Inversion of Ground Gravity and Airborne Gradient Data

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Society of Exploration Geophysicists, Houston, 2005

TOPICS

• Interaction between Euler Deconvolution Technique and 3D inversion

• Equivalence source technique vs FFT
  - e.g. derivatives, continuation

• Conclusion
Euler Deconvolution Technique - Review

**Euler’s Equation:**

\[
x_0 \frac{\partial G(x_i, y_i, z_i)}{\partial x} + y_0 \frac{\partial G(x_i, y_i, z_i)}{\partial y} + z_0 \frac{\partial G(x_i, y_i, z_i)}{\partial z} + NG_0 =
\]

\[
= NG(x_i, y_i, z_i) + x_i \frac{\partial G(x_i, y_i, z_i)}{\partial x} + y_i \frac{\partial G(x_i, y_i, z_i)}{\partial y} + z_i \frac{\partial G(x_i, y_i, z_i)}{\partial z}
\]

\[G(x_i, y_i, z_i)\] - measured data at locations, \(i = 1, 2, \ldots, M\)

\[G_0\] - constant background field

\[N\] - structural index (fall-off rate of the field \(G\))

\[ (x_0, y_0, z_0)\] - anomalous source

Gravity Structural Indices

0 - sill/dyke/step

0.5 - ribbon

1 - pipe

2 - sphere
Euler Deconvolution – solutions and post-processing

3D-Euler deconvolution technique (Reid et al. (1990), Zhang et al, 2000 )

Post-processing technique (Mikhailov et al. 2003 )
Forward Modeling - gravity and gradients

Quasi-Analytic Solution:

• Polyhedron of triangular facets
• Integrals by M. Okabe (1979).

\[
G_x(\vec{r}_0) = \gamma \iiint_Y \rho(\vec{r}) \frac{z - z_0}{|\vec{r} - \vec{r}_0|^3} \, d\nu
\]

\[
G_{xx}(\vec{r}_0) = 3\gamma \iiint_Y \rho(\vec{r}) \frac{(x - x_0)(z - z_0)}{|\vec{r} - \vec{r}_0|^5} \, d\nu
\]

\[
G_{xy}(\vec{r}_0) = 3\gamma \iiint_Y \rho(\vec{r}) \frac{(y - y_0)(z - z_0)}{|\vec{r} - \vec{r}_0|^5} \, d\nu
\]

\[
G_{xz}(\vec{r}_0) = \gamma \iiint_Y \rho(\vec{r}) \left[ \frac{3(z - z_0)^2}{|\vec{r} - \vec{r}_0|^3} - \frac{1}{|\vec{r} - \vec{r}_0|^3} \right] \, d\nu
\]

\[
\vec{r}_0 = (x_0, y_0, z_0) \text{ - observation location}
\]

\[
\vec{r} = (x, y, z) \text{ - source location}
\]

\[
V \text{ - volume of the anomalous mass}
\]

\[
\gamma \text{ - the gravitational constant}
\]
Synthetic Ground Gravity Survey

- 30 lines of length 2800m along the NS direction.
- Inline sampling 40m. The distance across lines is 100m.
- The station elevation is 1 m from ground surface.

Seg2005_1.dat
Salt Dome Model

Based on Peters and Dugan (1945)

Ground Surface

Cap Rock:
- density: 2.55 g/c.c.
- thickness: 18 m
- radius: 500 m

Salt:
- density: 2.2 g/c.c.
- depth: 1600 m

Host Rock:
- density: 2.67 g/c.c.
- thickness: 2000 m

Seg2005_1.dat
Computed Model - gradients of vertical gravity $G_z$

Synthetic across-line gradient

“True_dat”
a_dome

Seg2005_1
15.dat

1 E.O. (Eotvos) = 1e-4 mGal/m
Computed Model - gradients of vertical gravity Gz

True_data _dome

Seg2005_1 15.dat

Synthetic inline gradient
Computed Model - gradients of vertical gravity $G_z$

True_data_dome

Seg2005_115.dat

Synthetic vertical gradient
Euler Deconvolution- setting and data types

- **Euler Deconvolution Setting:**
  - Moving window size: 750 m by 750 m
  - Structural index range: 0.25 - 2.25
    (excluding 0 as there is no true anomaly with this rate)

- **3 types of data:**
  - *Quasi-analytic Gz with quasi-analytic gradients*
    interpolated on grid with cell size 20m by 50m (1/2 data)
  - *Quasi-analytic Gz and FFT gradients*
    FFT grid cell size 21m by 46 m
  - *Quasi-analytic Gz with noise and FFT gradients*
    FFT grid cell size 21m by 46 m
    Gaussian noise with S.D. = 0.05mgal (.1% of peak)
Euler Deconvolution - Synthetic VS. FFT Gradients

Red – synthetic quasi-analytic solution.
Blue – FFT using Gz (no noise).
Green - FFT using Gz (noise).

Across-line gradient along a NS profile at east –27m.

Seg2005_125.dat, Seg2005_124.dat, Seg2005_126.dat
Euler Deconvolution - Synthetic VS. FFT Gradients

Red – quasi-analytic solution.
Blue – FFT using Gz (no noise).
Green - FFT using Gz (noise).

InLine Gradient of Gz at x=-27

tighter inline grid due to higher inline sampling

Seg2005_125.dat, Seg2005_124.dat, Seg2005_126.dat
Euler Deconvolution - Synthetic VS. FFT Gradients

Vertical Gradient of Gz at x=27

- Red – quasi-analytic solution
- Blue - FFT using Gz (no noise)
- Green - FFT using Gz (noise)

incorporates errors of crossline

Seg2005_125.dat, Seg2005_124.dat,
Seg2005_126.dat
Euler Solutions-comparison

Note: after post-processing

Quasi-Analytic Gz with Quasi-Analytic gradients
Quasi-Analytic Gz with FFT gradients
Quasi-Analytic Gz with noise and FFT gradients

Colors indicate the deviation of solution

Red: low, Yellow: medium, Blue: high

Seg2005_90.dat, Seg2005_88.dat, Seg2005_93.dat
How to generate gradients effectively?

As seen, it is essential to have accurate gradients in order to generate good Euler solutions.

The problem is that FFT gradients are not reliable in the case that the line spacing is “large”.

Next we will demonstrate that accurate gradients can be generated utilizing 3D-inversion (reference???).
3D-Gravity Inversion-theory

Inversion Model

Minimization Technique: Conjugate Gradient

Minimize \( \| \phi - \phi^\star \| \)

Subject to \( l_i \leq \rho_i \leq u_i, \ i = 1, 2, \ldots, N \)

\[ \phi = \| W_d (d - \hat{d}) \|^2 \]

\[ d = d(\rho_1, \rho_2, \ldots, \rho_M) \]

\( \phi^\star \) prescribed tolerance

\( (\hat{d}_1, \hat{d}_2, \ldots, \hat{d}_M) \) measured data

\( (d_1, d_2, \ldots, d_M) \) simulated data

\( W_d = \text{diag}(1/\sigma_1, 1/\sigma_2, \ldots, 1/\sigma_M) \)

\( \sigma_i \) is the standard deviation of the \( i \)-th datum

3D-volume

\textit{size ?}, \textit{depth ?}
3D-Gravity Inversion - synthetic data

- Apply inversion to Gz of salt dome
- Add random noise to Gz
  - \( \sigma = 0.05 \text{ mGal} \)
- Initial 3D-grid volume
  - dimensions = 3200 m x 3000 m x 2000 m \((l_x, l_y, l_z)\)
- The distance from top of the 3D-volume to measurement points
  - 50 m.
- Grid cell
  - width 100m in easting, 50m in northing, and vertical extent 100m
- Utilize the data deviation during the inversion - \( \chi^2 \)
3D-Gravity Inversion-inverted model

Gz inversion

Section at X=0

Inversion is too shallow!
3D-Gravity Inversion - misfit

Computer Gz with Noise

Gz inversion misfit  ie. relatively random.

Seg2005_102.dat  Seg2005_115.dat
Euler Deconvolution—utilization of inversion results

EQUIVALENT SOURCE TECHNIQUE

Generate Gz and its gradients from inverted model

- Apply Euler Deconvolution to these data
  - Moving window size: 750 m by 750 m
  - Structural index range: 0.25 - 2.25

Objective: Generate more accurate Euler solutions as starting model for inversion.
Comparison of Gradients - along NS profile at easting –27m.

Red – Computed Solution.
Blue - Equivalence Source - i.e. computed from inversion model using Gz (noise)
Green – FFT using Gz (noise).
Euler Solutions - comparison

Conclusion:
Euler solutions can be improved with the use of inverted gradients

Seg2005_93.dat, Seg2005_111.dat, Seg2005_90.dat
3D-Gravity Inversion - Utilization of Euler solution information

Gz Inversion

Cap: 2.55 g/c.c.
Salt: 2.2 g/c.c
Background: 2.67 g/c.c

X=0 Section of Inverted model

Seg2005_104.dat
3D-Gravity Data Inversion - misfit

The top of 3D-grid volume is 300 m from ground surface.

Gz inversion misfit.

Seg2005_107.dat  Seg2005_115.dat
Upward continuation – utilization of 3D-Inversion

**Purpose:** to demonstrate that equivalent source technique can be used for upward continuation

1. Compute the responses of the dome at 80m above the ground using
2. perform upward continuation by standard FFT
3. perform upward continuation by Equivalent source technique
4. Compare with analytic solutions
Upperward continuation – utilization of 3D-Inversion


Along NS profile at x=1500m Flight height: 80m
Upperward continuation – utilization of 3D-Inversion

Along NS profile at x=27m.  Flight height: 80m
Upward continuation – utilization of 3D-Inversion

Along NS profile at x=1500m  Flight height: 80m


Gzx @ 80m

Seg2005_129.dat, Seg2005_130.dat, Seg2005_128.dat
Upperward continuation – utilization of 3D-Inversion

Along NS profile at x=27m.  Flight height: 80m


Seg2005_129.dat, Seg2005_130.dat, Seg2005_128.dat
Based on inverted models, various components of gravity/gradient data can be generated accurately by the equivalent source technique.

- Gravity/gradient data generated with inverted models can greatly enhance the Euler Deconvolution technique.

- Euler solutions can provide useful information for setting starting models of the inversion.

- Utilization of data deviation gives better inversion results.

- The equivalent source technique can provide better results than standard FFT for derivatives and continuation (filtering?)
Test equivalent source technique with real airborne gradient data.