Some Pitfalls
in
Gradient Magnetic Processing

or

Why Rotate Gradients

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Example 1

3 Sensor Magnetic Processing:
Derivation of TMI Gradients

Algorithms for de-rotating

2 horizontally offset magnetometers

with 1 vertically offset magnetometer

In this configuration, the 3 sensors are set on a rigid frame which varies its orientation continuously during flight. The problem is how to obtain gradients in some useful and consistent coordinate system.

Locally (ie. at each position), the sensors measure derivatives in somewhat random orientations. Unless, the gradients can be de-rotated to a consistent frame then they have limited usefulness.

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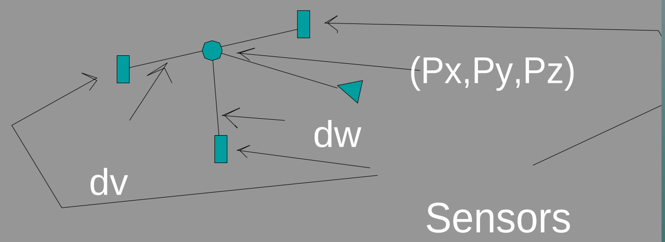
3 TMI measurements at each station in the local frame (platform) originating at (Px,Py,Pz)



$$\begin{cases} M_1 = M(P_x, P_y - \Delta v, P_z) & \text{(port)} \\ M_3 = M(P_x, P_y + \Delta v, P_z) & \text{(starboard)} \\ M_2 = M(P_x, P_y, P_z - \Delta w) & \text{(lower)} \end{cases}$$

"Global" vs. Platform (local) Frame

Normally gradient vectors can be orientated easily from a local frame to a more general geographic, geomagnetic or grid system if the orientation of the rigid system and the 3 gradient vectors are known. Mathematically,

(u,v,w) platform

(x,y,z) Grid , Geomagnetic, Geographic

$$\frac{\partial M}{\partial x} = \frac{\partial M}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial M}{\partial v}\frac{\partial v}{\partial x} + \frac{\partial M}{\partial w}\frac{\partial w}{\partial x}$$

$$\frac{\partial M}{\partial y} = \frac{\partial M}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial M}{\partial v}\frac{\partial v}{\partial y} + \frac{\partial M}{\partial w}\frac{\partial w}{\partial y}$$

$$\frac{\partial M}{\partial z} = \frac{\partial M}{\partial u}\frac{\partial u}{\partial z} + \frac{\partial M}{\partial v}\frac{\partial v}{\partial z} + \frac{\partial M}{\partial w}\frac{\partial w}{\partial z}$$

The orientation is represented mathematically by:

$$\frac{\partial(u,v,w)}{\partial(x,y,z)}$$

i.e. pitch, roll, heading

Two of the platform derivatives derived via the following equations:

$$\frac{\partial M}{\partial v} = \frac{M_3 - M_1}{2\Delta v}$$

Platform Transverse Gradient

$$\frac{\partial M}{\partial w} = \frac{\frac{1}{2}(M_1 + M_3) - M_2}{\Delta w}$$

Platform Vertical Gradient

At each data point, we wish to recover the 3 gradients in the grid system. i.e., $\delta M/\delta x$, $\delta M/\delta y$, and $\delta M/\delta z$. However,

$\Rightarrow$ three equations in four unknowns

$$\frac{\partial M}{\partial u}, \frac{\partial M}{\partial x}, \frac{\partial M}{\partial y}, \frac{\partial M}{\partial z}$$

!!??

In-Line Derivative

The derivative in the direction of flight (in-line) has no direct measurement for this configuration. Normally, it is estimated by the difference in 2 in-line average measurements, i.e.

$$\frac{\partial M}{\partial u} \approx \frac{\overline{M}_1(x_1, y_1, z_1) - \overline{M}_2(x_2, y_2, z_2)}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 - (z_1 - z_2)^2}}$$

Where, $\overline{M}_{1,} \overline{M}_2$ are the average M at each data point.

However, using this estimate of the in-line derivative with the other local derivatives to de-rotate the gradients as if in a fixed rigid system presents problems. *As will be seen later*.

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Directional Derivative

Another approach is to use the notion of a directional derivative

Flight Path

$$A = (P_x, P_y, P_z) - (P_x', P_y', P_z')$$

Previous observation at (Px′,Py′,Pz′)

$$A_x \frac{\partial M}{\partial x} + A_y \frac{\partial M}{\partial y} + A_z \frac{\partial M}{\partial z} = \overline{M} - \overline{M}'$$

$$\begin{cases} \overline{M} = \frac{1}{3}(M_1 + M_2 + M_3) \\ \overline{M}' = \frac{1}{3}(M_1' + M_2' + M_3') \end{cases}$$

fourth equation !

Fully Synthetic Data Example

Simulated Flight Path

- **roll, pitch, heading, altitude variations and thus magnetometer frame orientations generated semi-randomly**
- **Simulated data mathematically generated allowing analytic gradients for benchmarking**

•In this example, the locations of the center of the sensors vary pseudo-randomly along a prescribed flight path and the orientations of the 3-sensor rigid system are allowed to vary at each data position in a smooth but random fashion. The data at each sensor is generated by simulation at the location of that sensor by means of *PetRos EiKon's* 3D Magnetic modeling functions.

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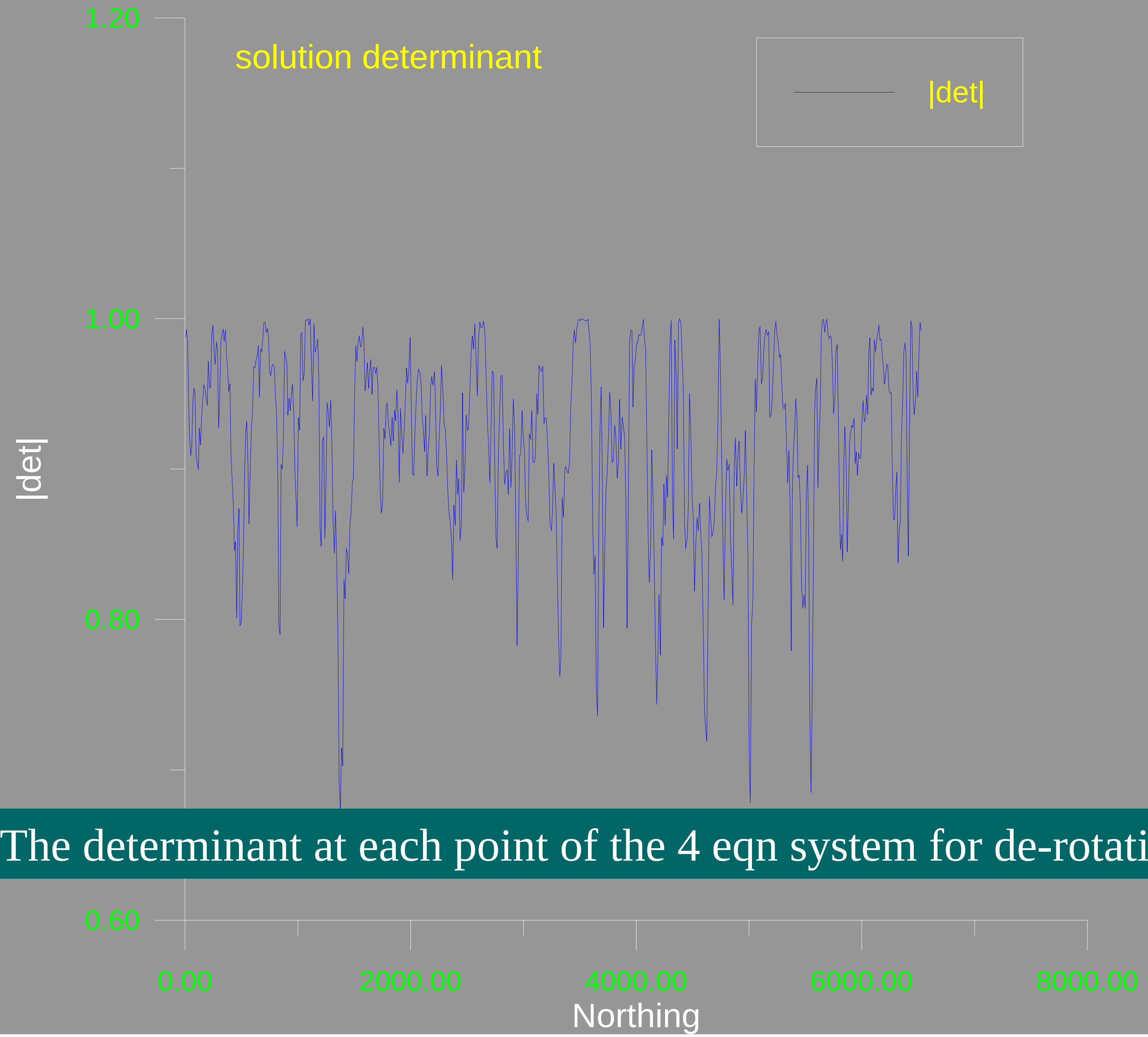
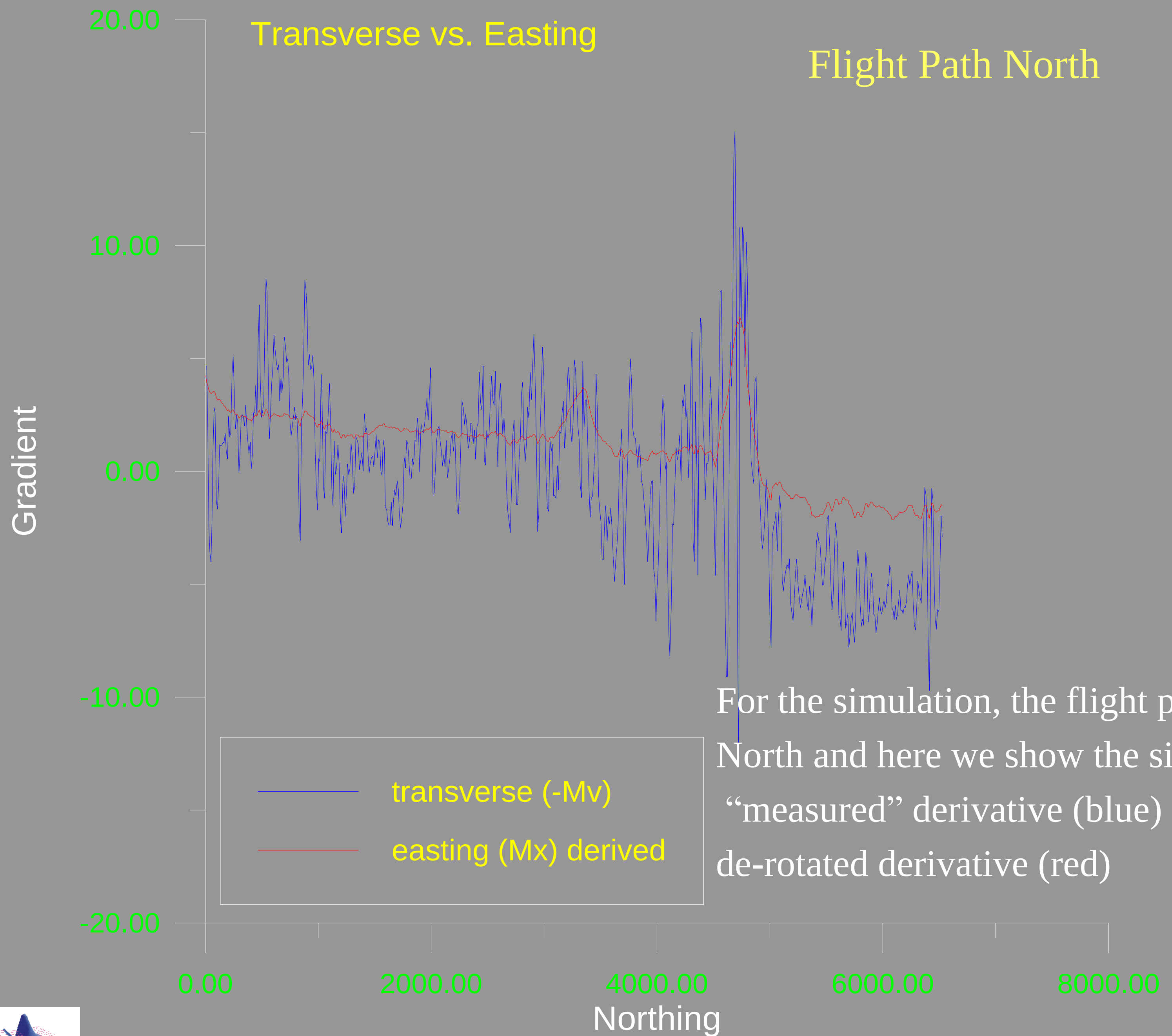
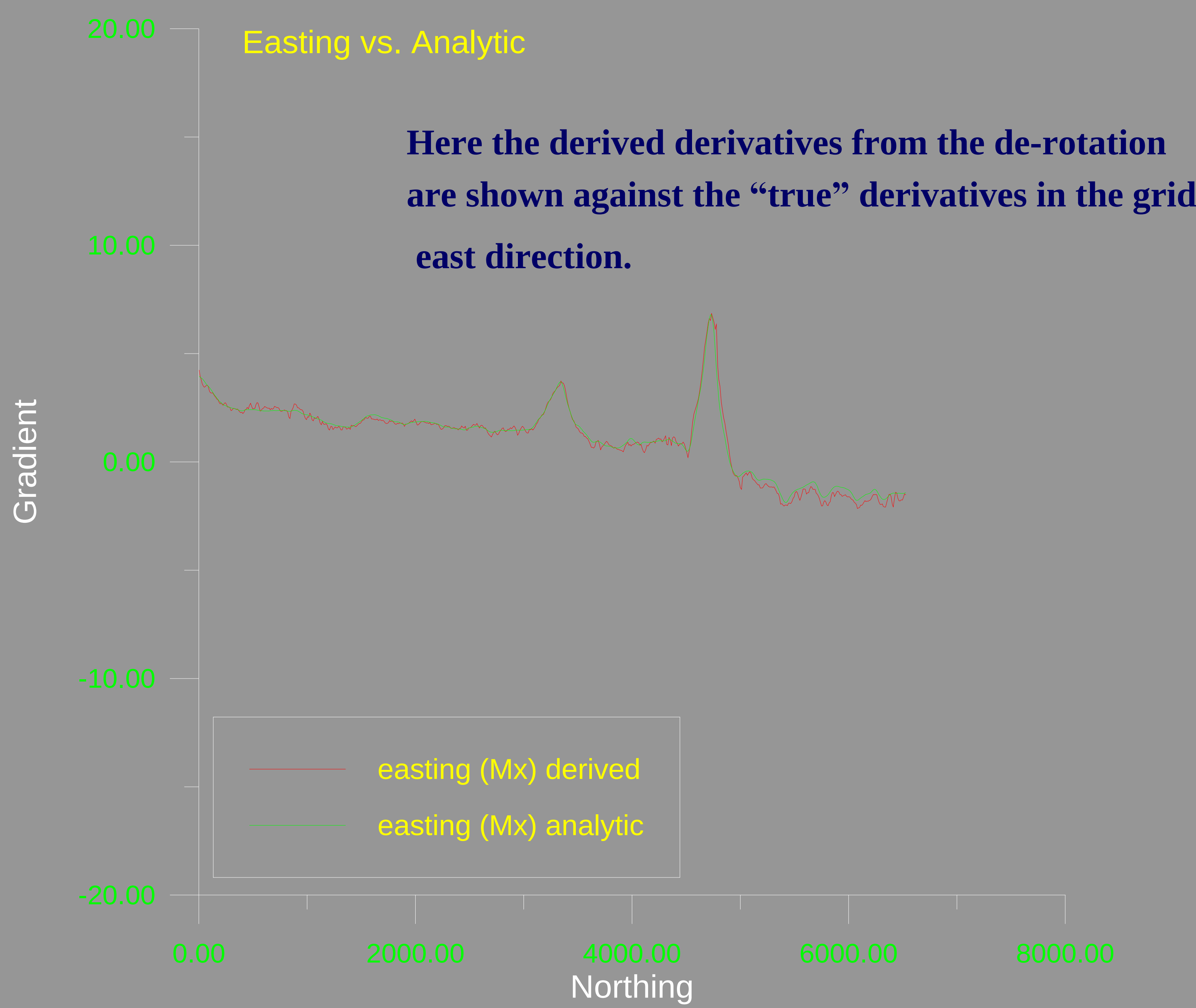


The determinant at each point of the 4 eqn system for de-rotation

Flight Path North



For the simulation, the flight path is generally North and here we show the simulated transverse “measured” derivative (blue) against the de-rotated derivative (red)


20.00
Easting vs. Analytic
Here the derived derivatives from the de-rotation are shown against the “true” derivatives in the grid east direction.
10.00
Gradient
0.00
-10.00
easting (Mx) derived
easting (Mx) analytic
-20.00
0.00
2000.00
4000.00
6000.00
8000.00
Northing

Synthetic (simulated) data example with True flight and orientation information

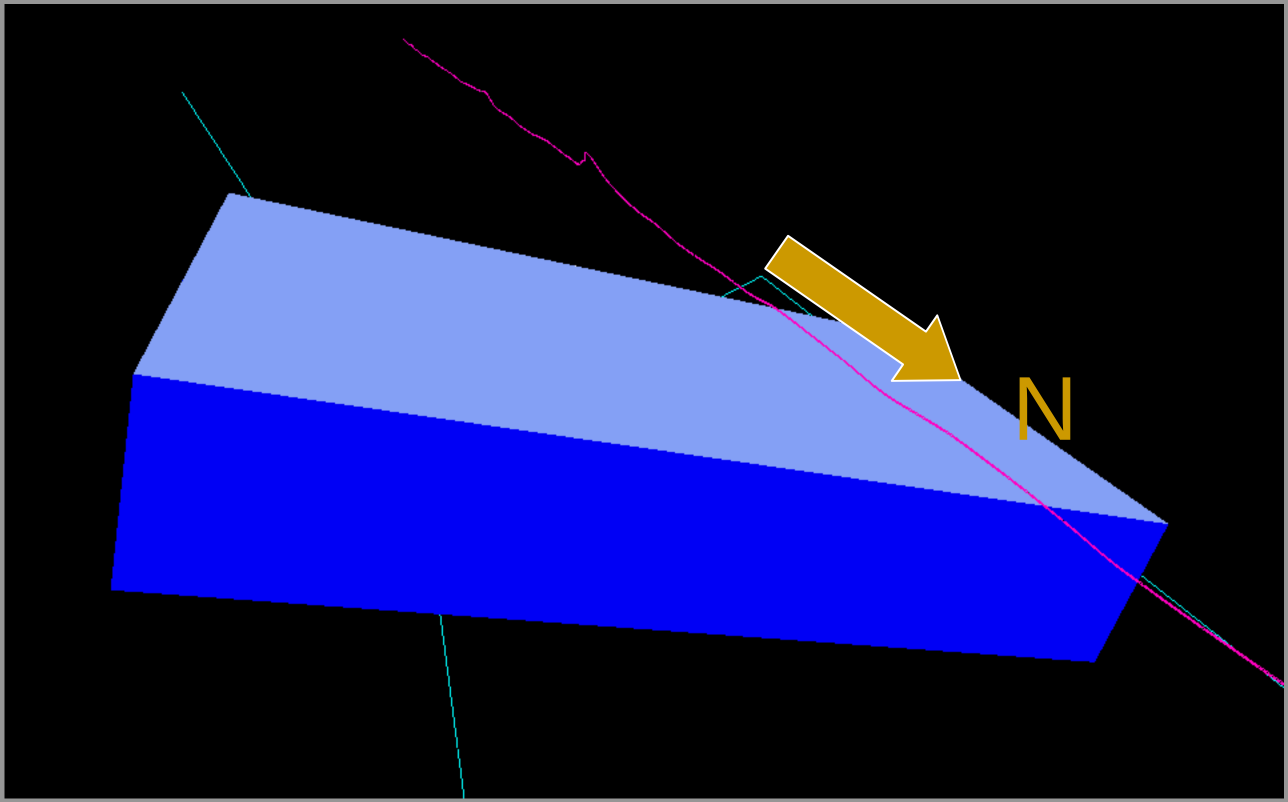
Section of a Single survey line (LINE10) with approximately 620 stations

Simulated data is generated on three profiles which follow the trajectories of the three sensors based on given flight path and orientation information

> (x,y,z), pitch, roll, heading are from real data

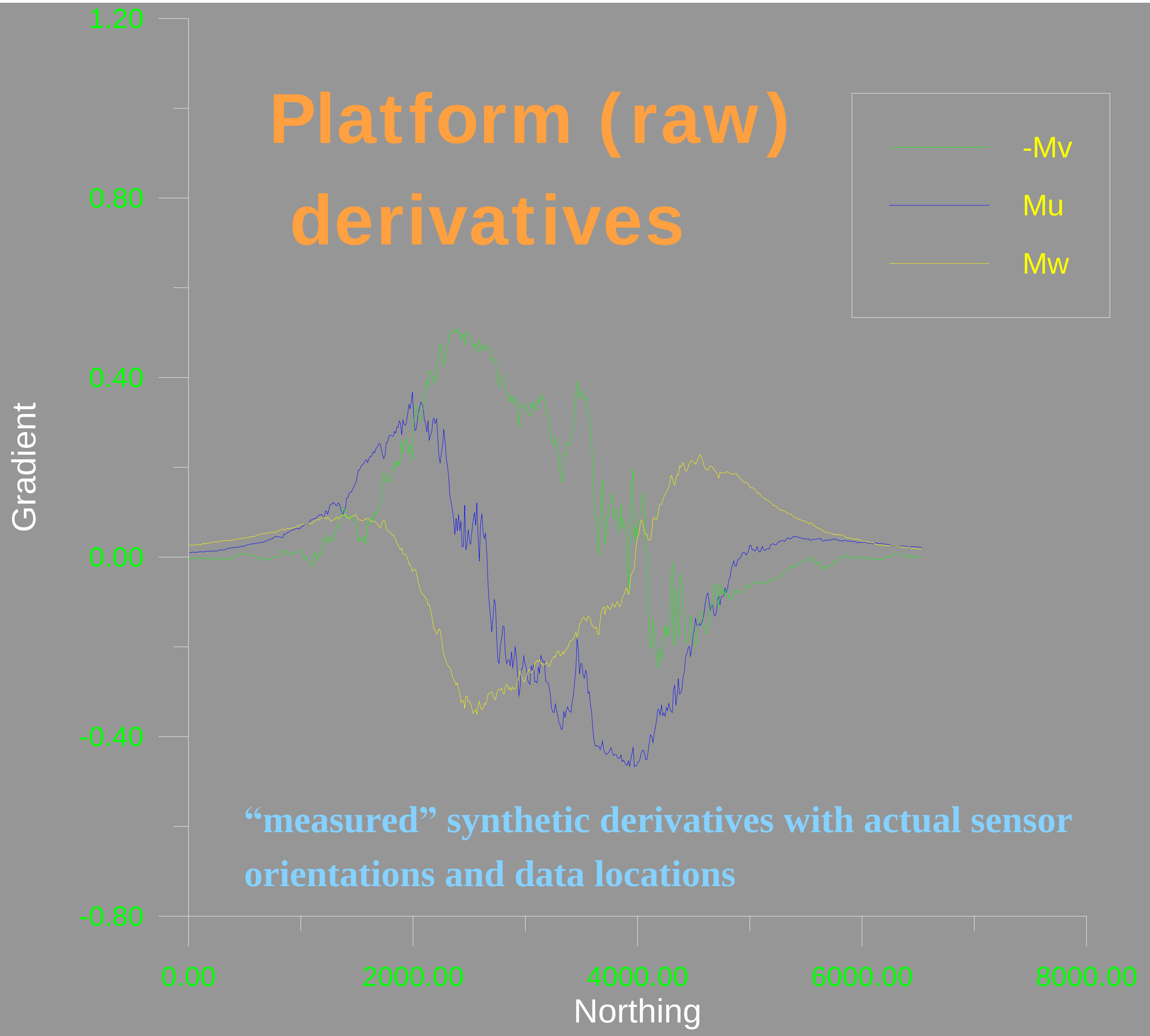
For this example, we utilize actual data locations and sensor orientations but use synthetic data to check the processing technique

Magnetic prism model

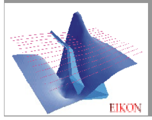


Here we show the synthetic model and the actual flight path with elevation for synthesis.

Platform (raw) derivatives

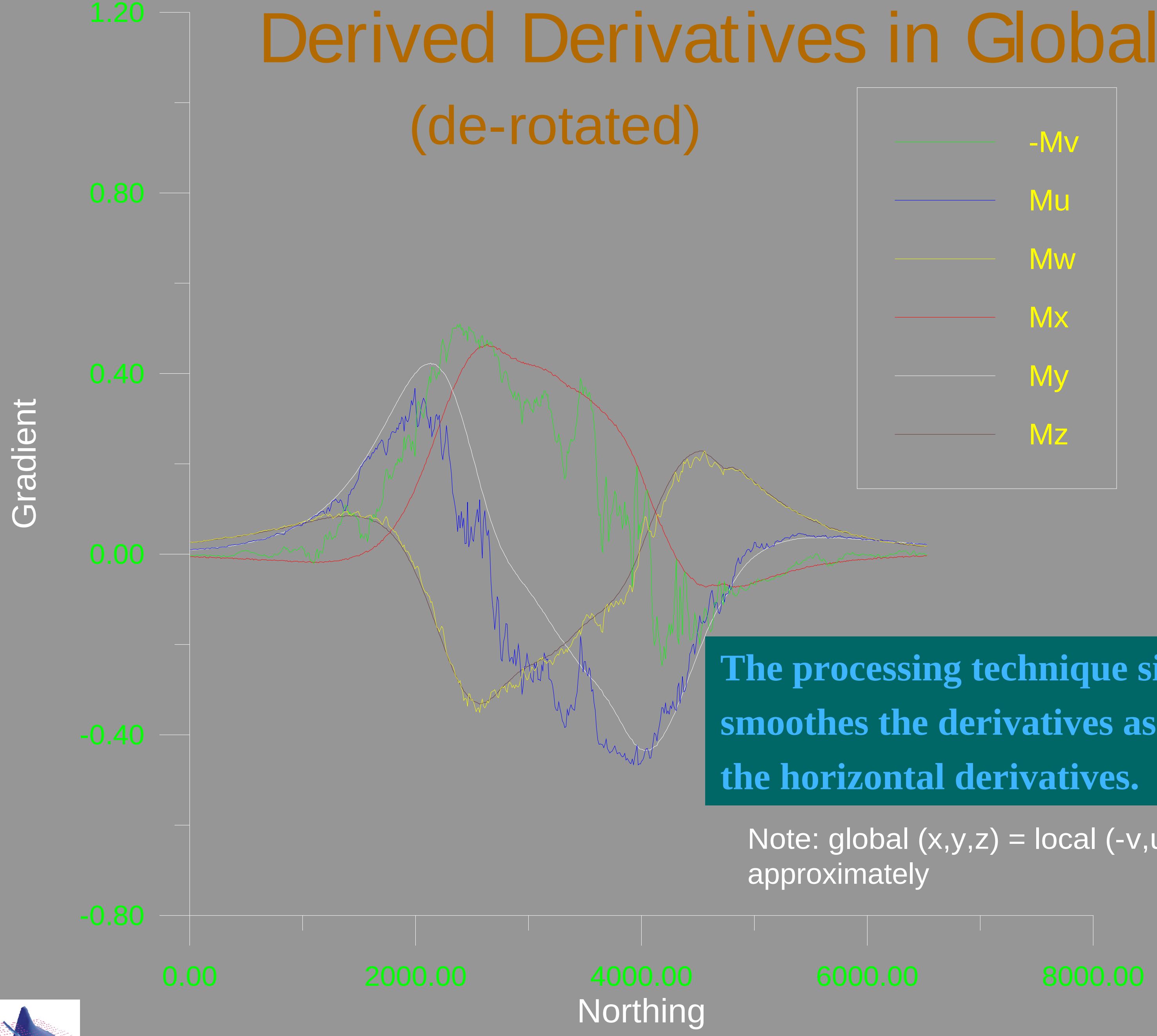


“measured” synthetic derivatives with actual sensor orientations and data locations



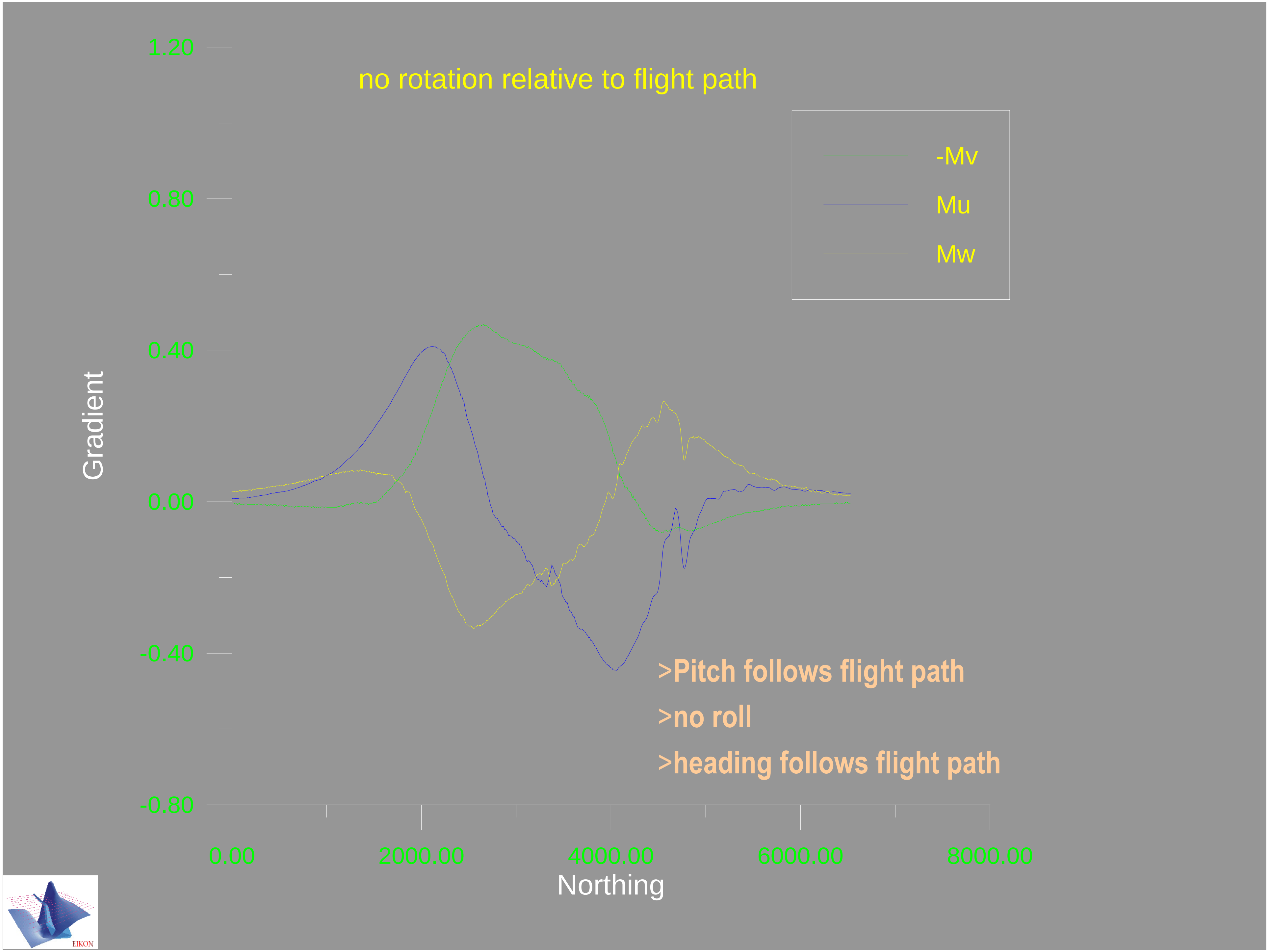
Derived Derivatives in Global System

(de-rotated)



The processing technique significantly smoothes the derivatives as well as un-mixing the horizontal derivatives.

Note: global (x,y,z) = local (-v,u,w) approximately



no rotation relative to flight path
-Mv
Mu
Mw
1.20
0.80
0.40
0.00
-0.40
-0.80
Gradient
0.00
2000.00
4000.00
6000.00
8000.00
Northing
>Pitch follows flight path
>no roll
>heading follows flight path
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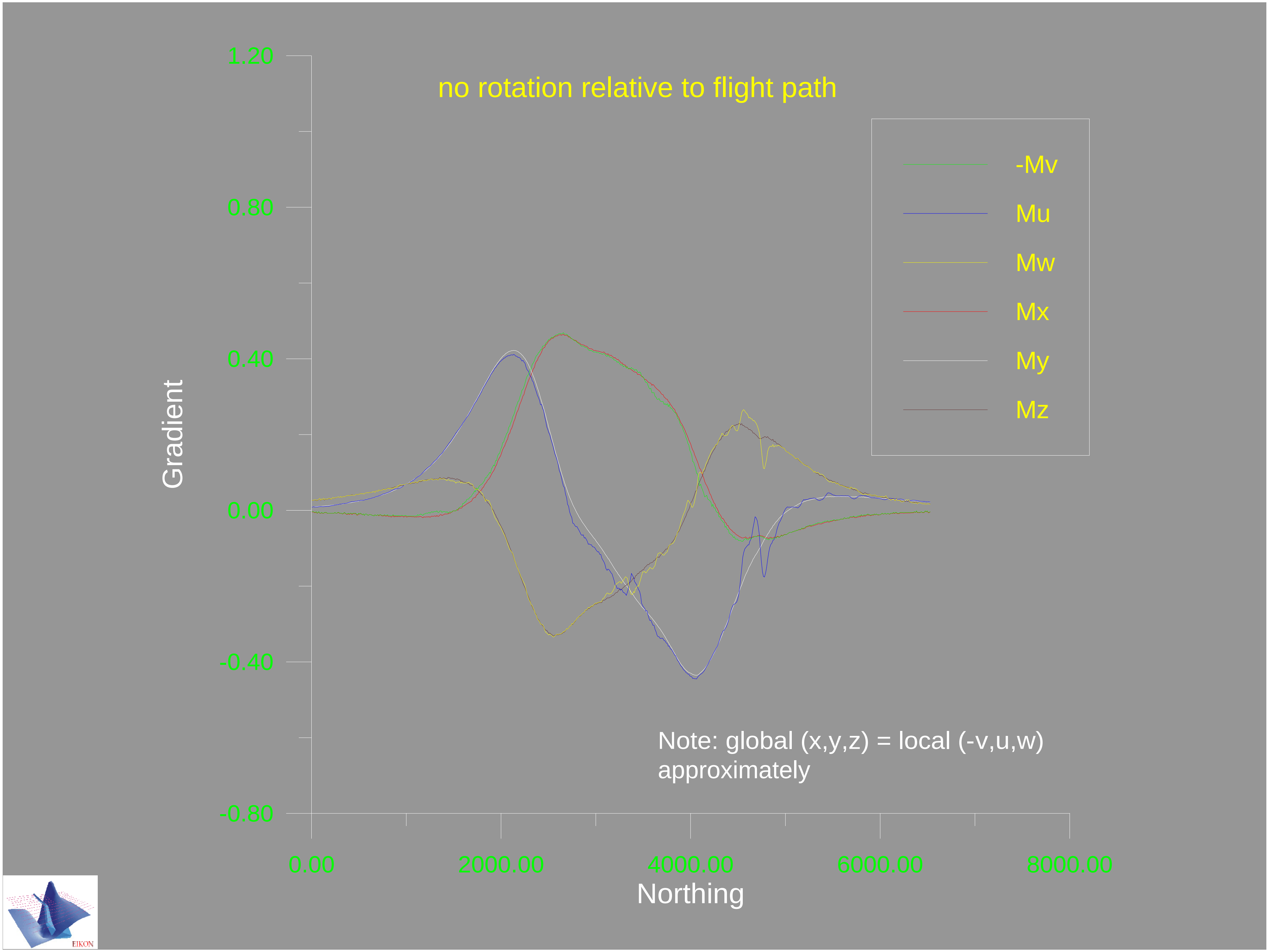

no rotation relative to flight path
-Mv
Mu
Mw
Mx
My
Mz
Gradient
1.20
0.80
0.40
0.00
-0.40
-0.80
0.00
2000.00
4000.00
6000.00
8000.00
Northing
Note: global (x,y,z) = local (-v,u,w)
approximately

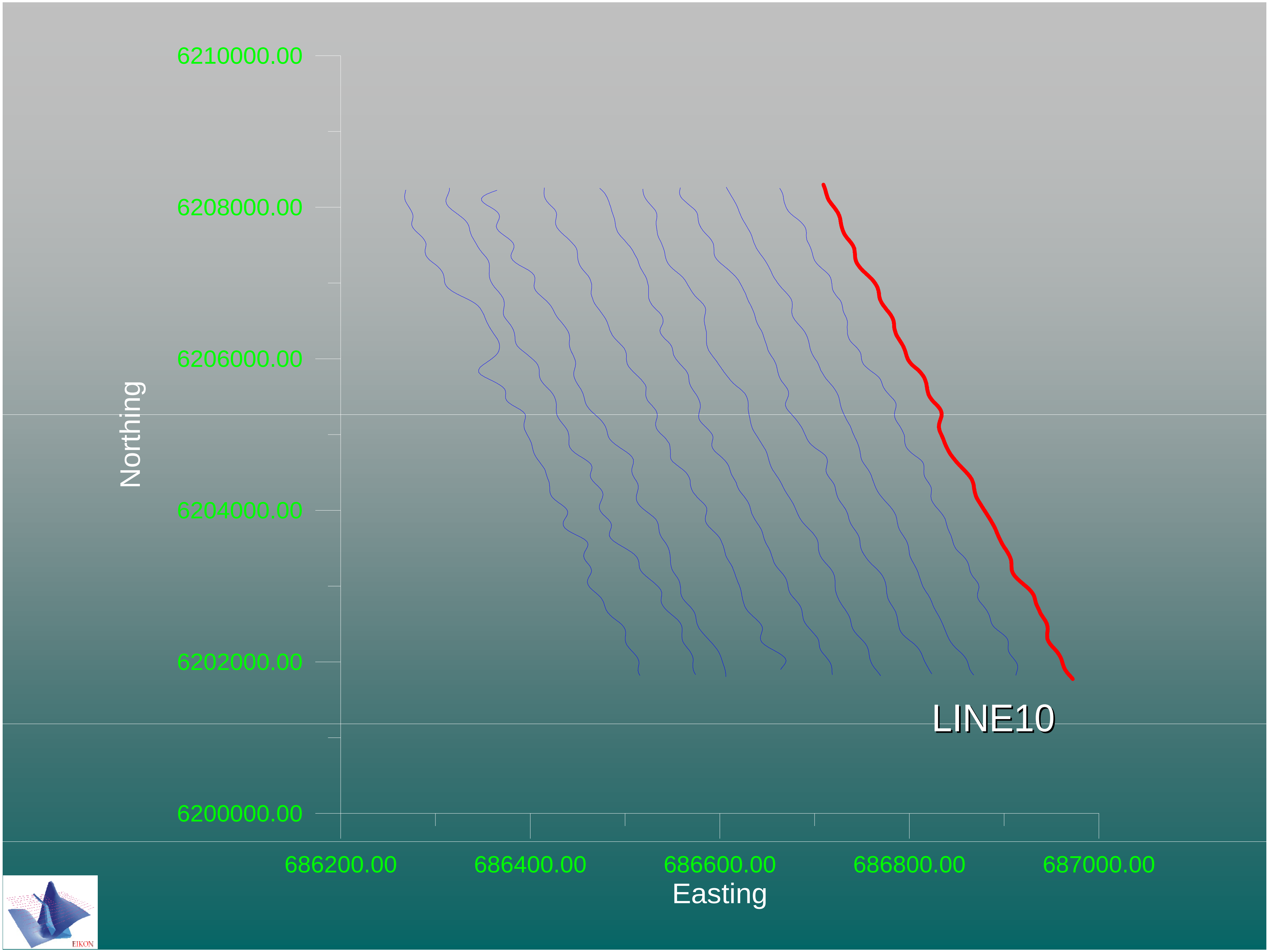
Gradient derivation from real data

Measured "corrected" data

– (LINE10, LINE70)

Processed to derivatives based on given flight path and orientation information

Here we examine several effects of examining or using derivatives without correct processing



6210000.00
6208000.00
6206000.00
6204000.00
6202000.00
6200000.00
Northing
LINE10
686200.00
686400.00
686600.00
686800.00
687000.00
Easting
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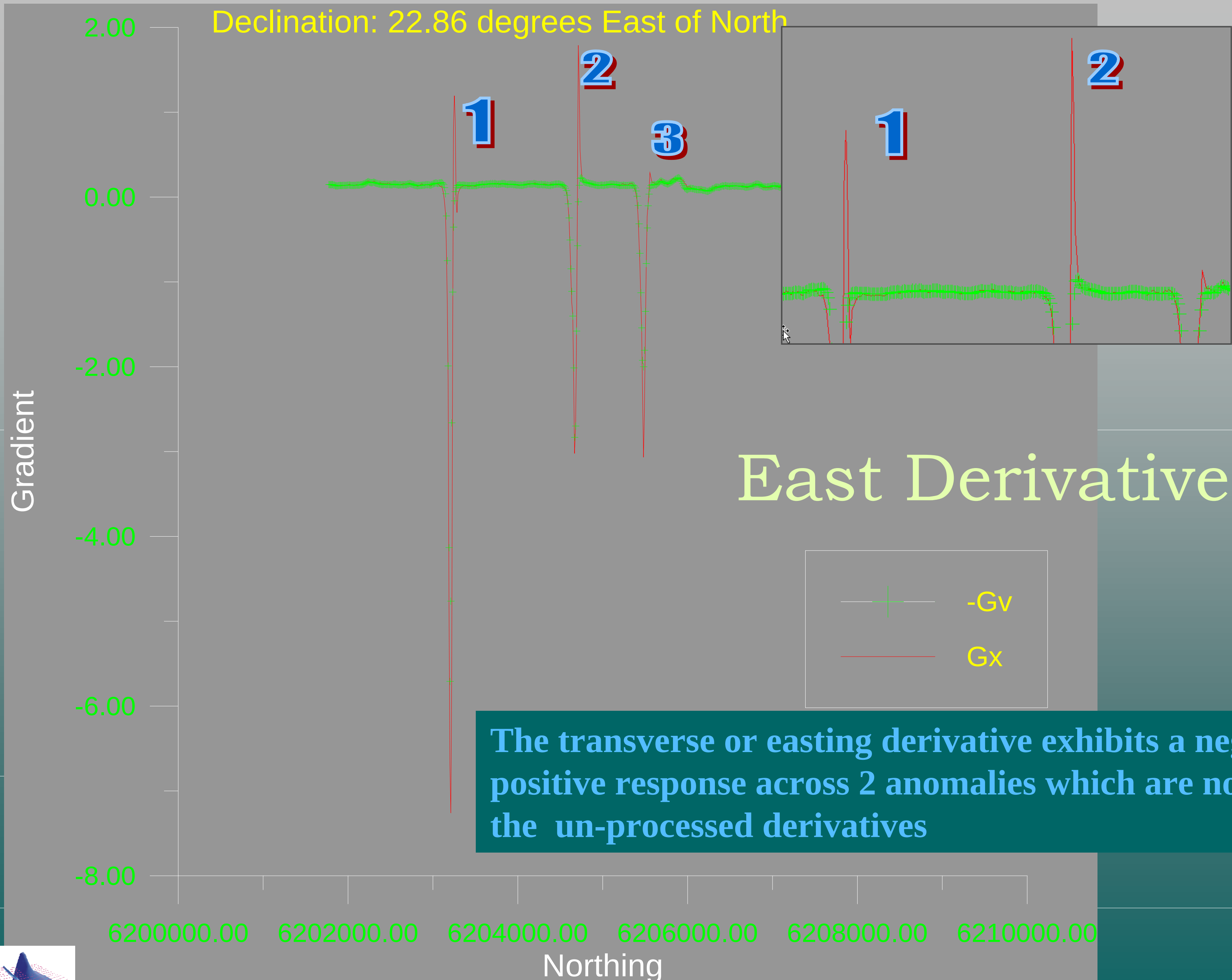
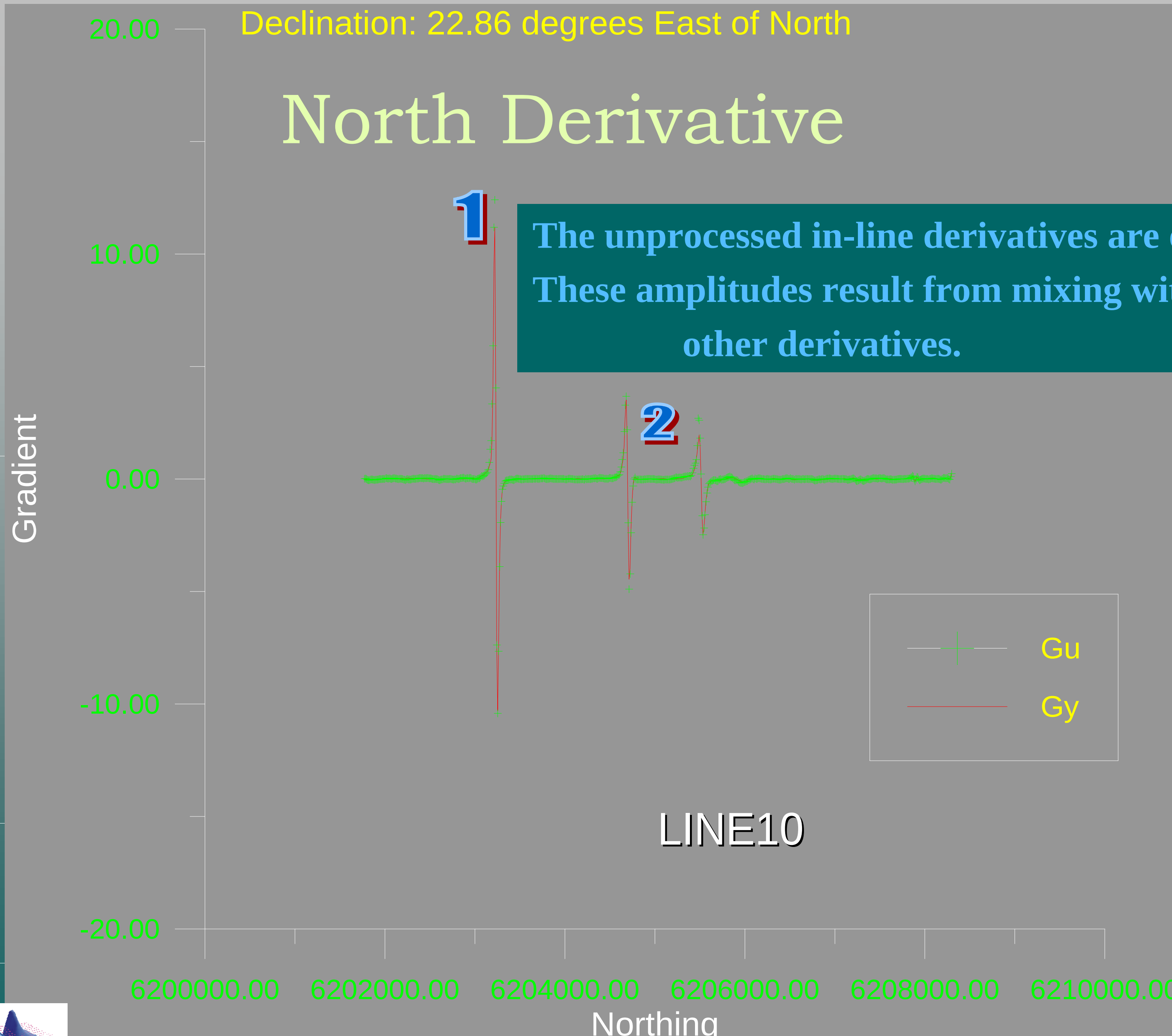
East Derivative

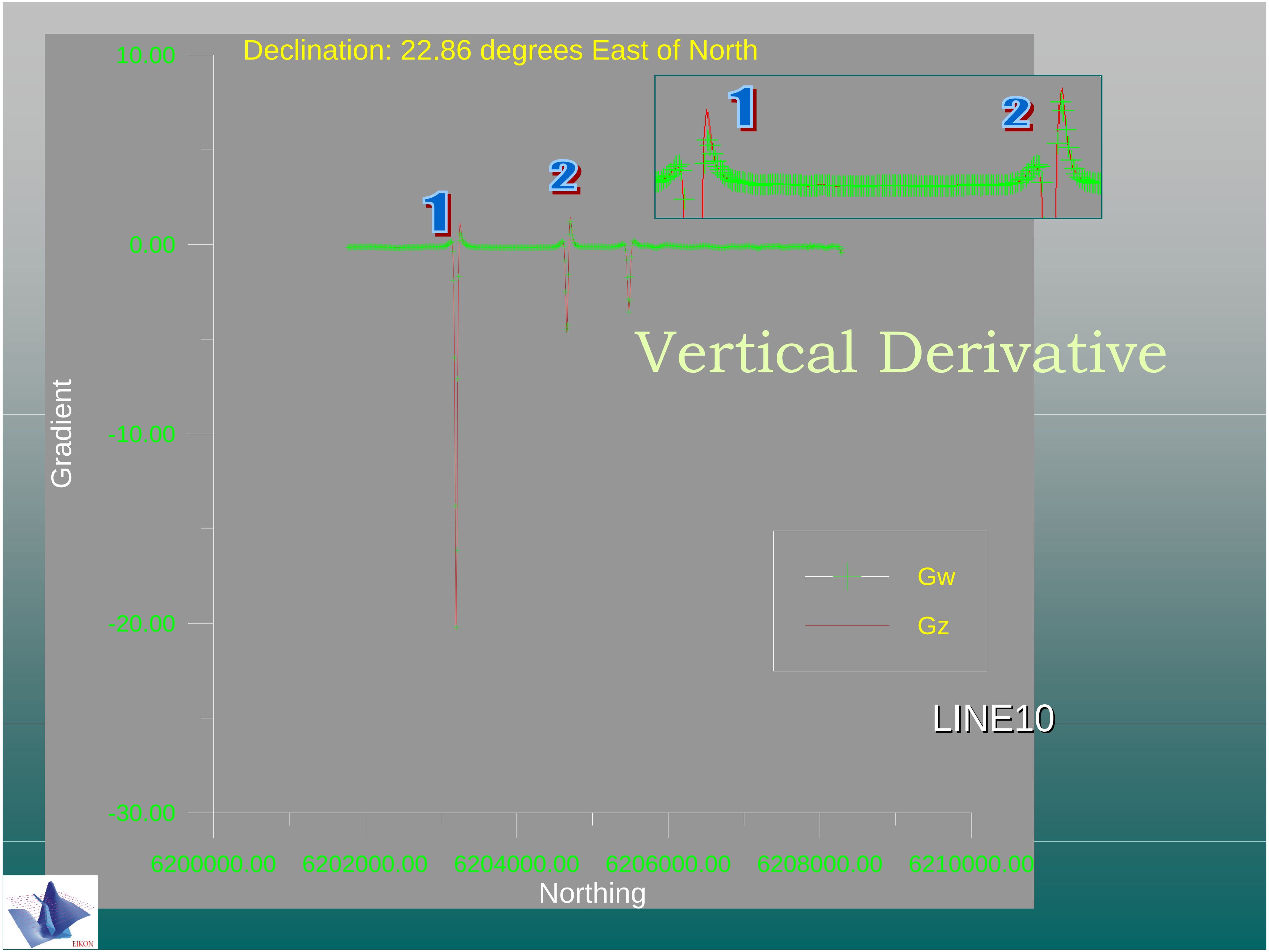


The transverse or easting derivative exhibits a negative-positive response across 2 anomalies which are not seen in the un-processed derivatives

North Derivative

The unprocessed in-line derivatives are overestimated. These amplitudes result from mixing with the other derivatives.



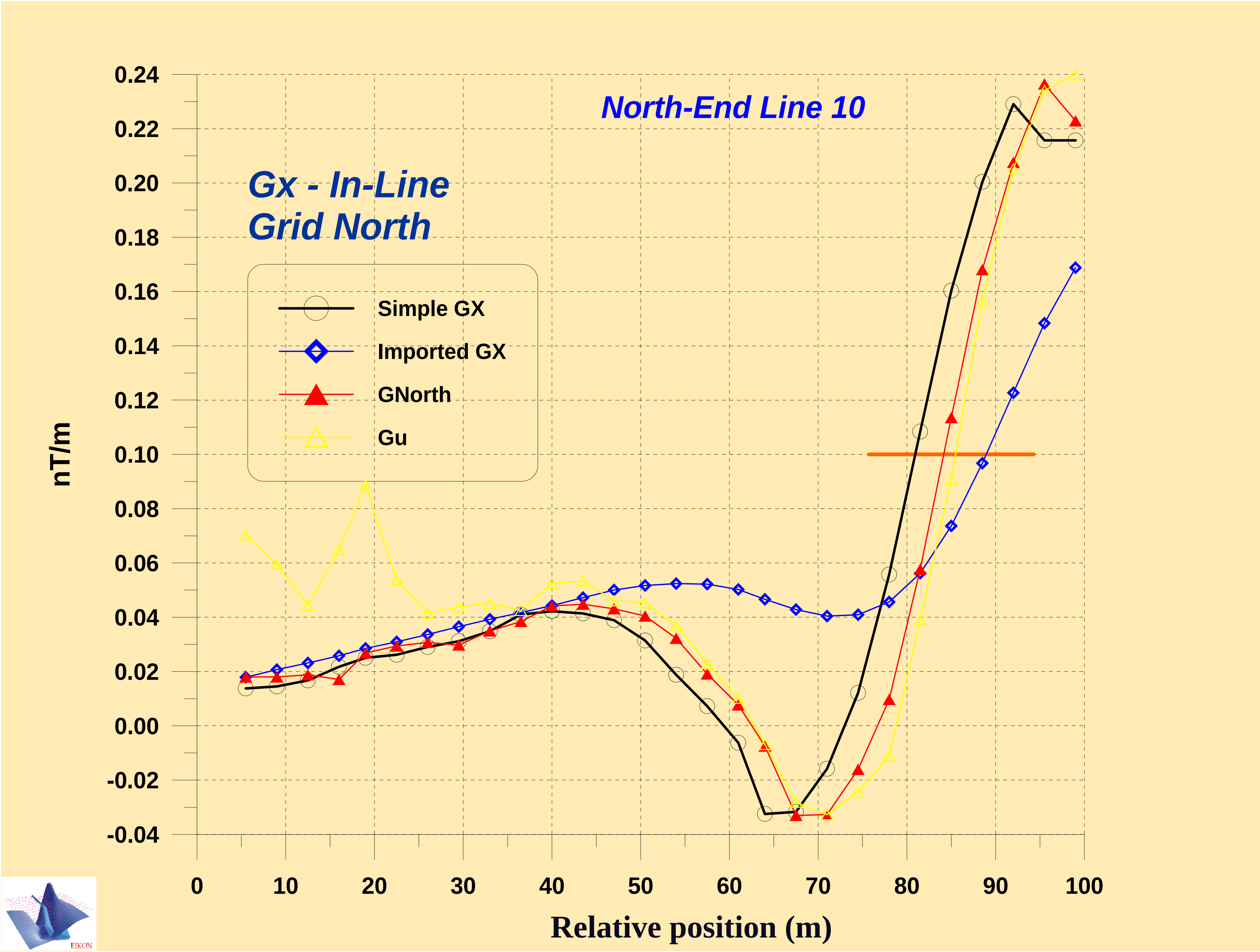


Declination: 22.86 degrees East of North
Vertical Derivative
10.00
0.00
-10.00
-20.00
-30.00
Gradient
1
2
Gw
Gz
LINE10
6200000.00
6202000.00
6204000.00
6206000.00
6208000.00
6210000.00
Northing
EIKON

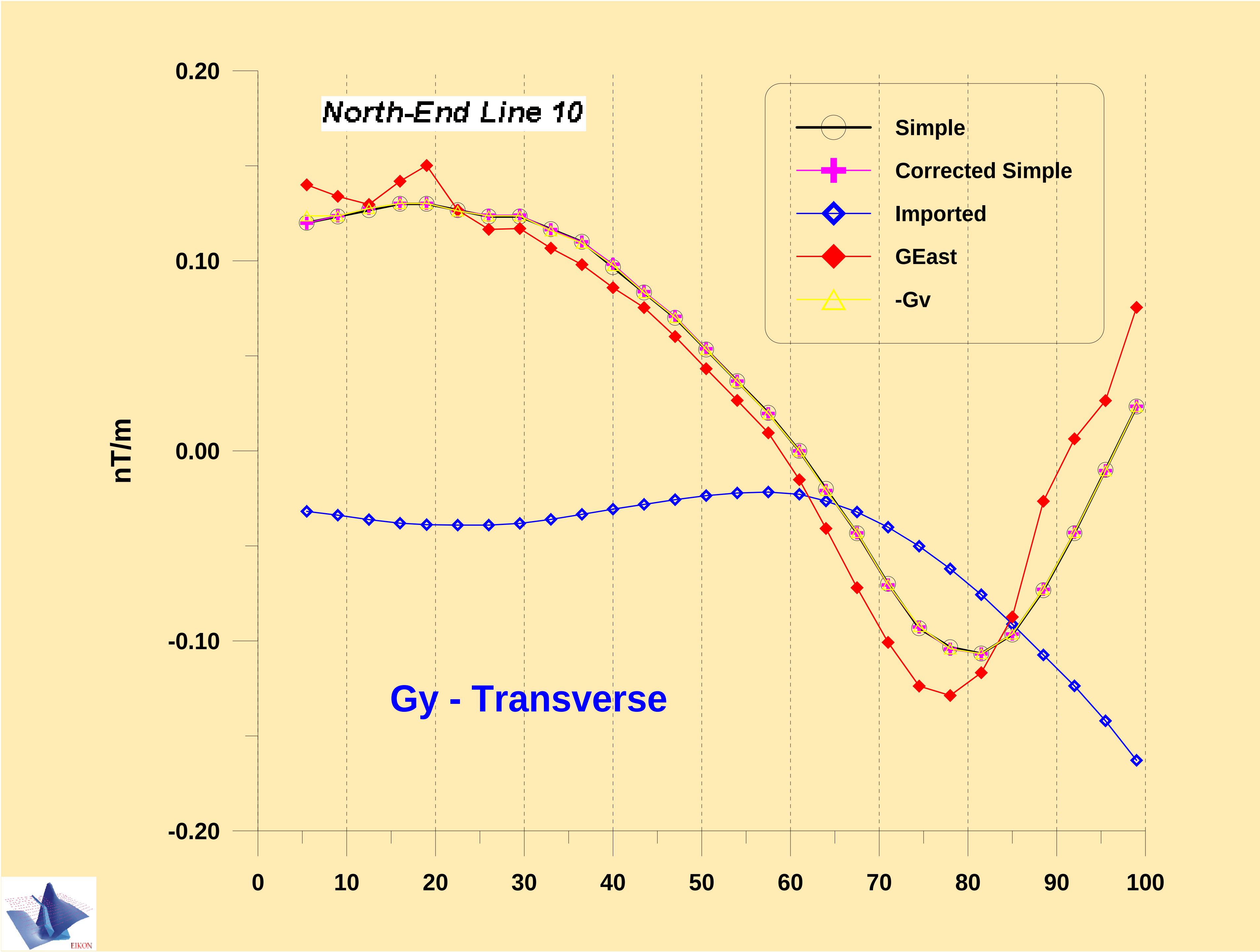
Correct Positioning of Derivatives

•In the following figures;

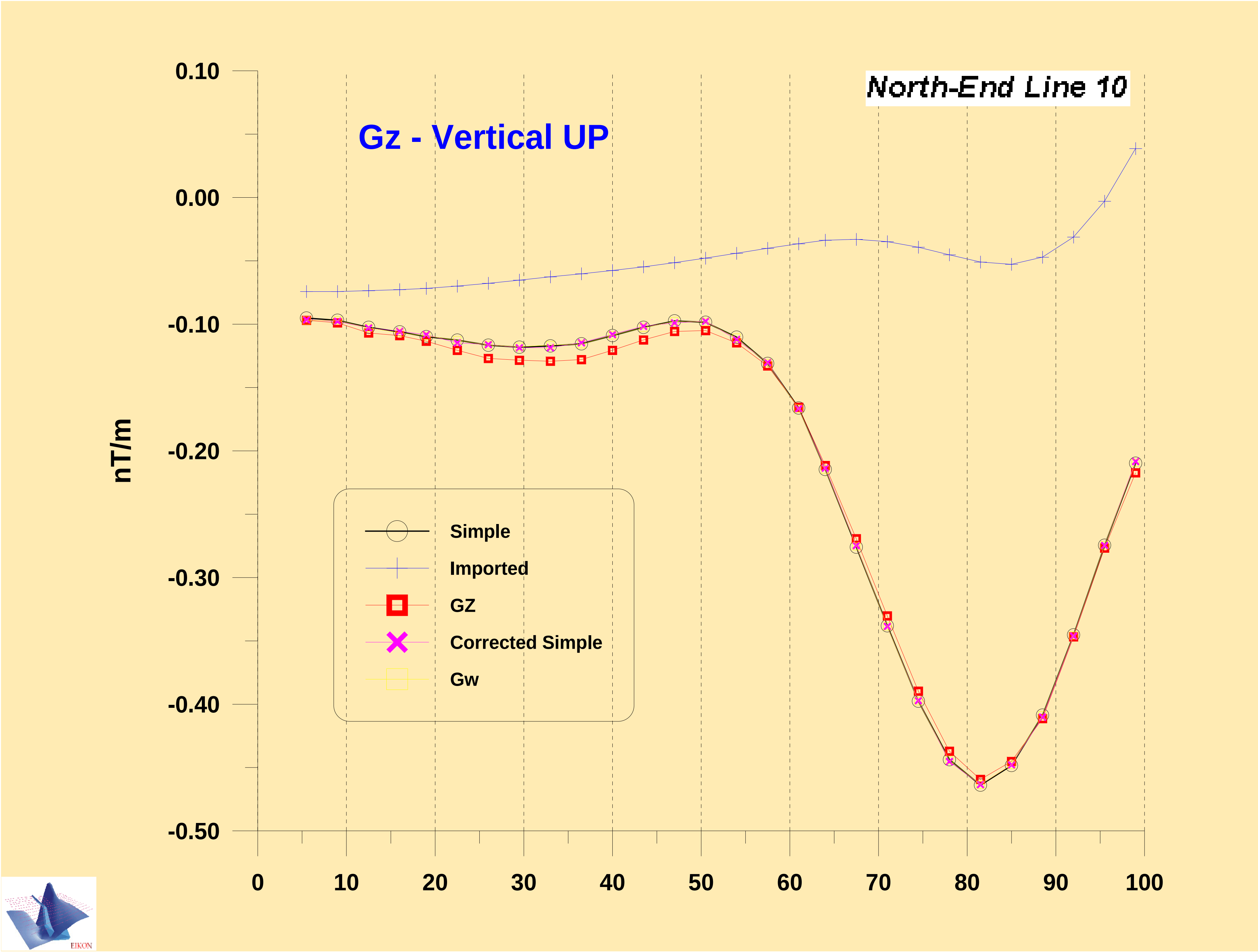
- Δ - Instantaneous collected derivative prior to processing
- O - Simple rotation of in-line based on measured derivatives being in a fixed frame
- ◇ - Alternative processing
- Δ - derivative in global (grid) frame – de-rotated derivatives
- \+ - Alternative simple rotation technique



North-End Line 10
Gx - In-Line
Grid North
Simple GX
Imported GX
GNorth
Gu
nT/m
0.24
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
-0.02
-0.04
0
10
20
30
40
50
60
70
80
90
100
Relative position (m)



North-End Line 10
Simple
Corrected Simple
Imported
GEast
-Gv
Gy - Transverse
nT/m
0.20
0.10
0.00
-0.10
-0.20
0
10
20
30
40
50
60
70
80
90
100



North-End Line 10
Gz - Vertical UP
nT/m
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
0
10
20
30
40
50
60
70
80
90
100
Simple
Imported
GZ
Corrected Simple
Gw
EIKON

Example 2

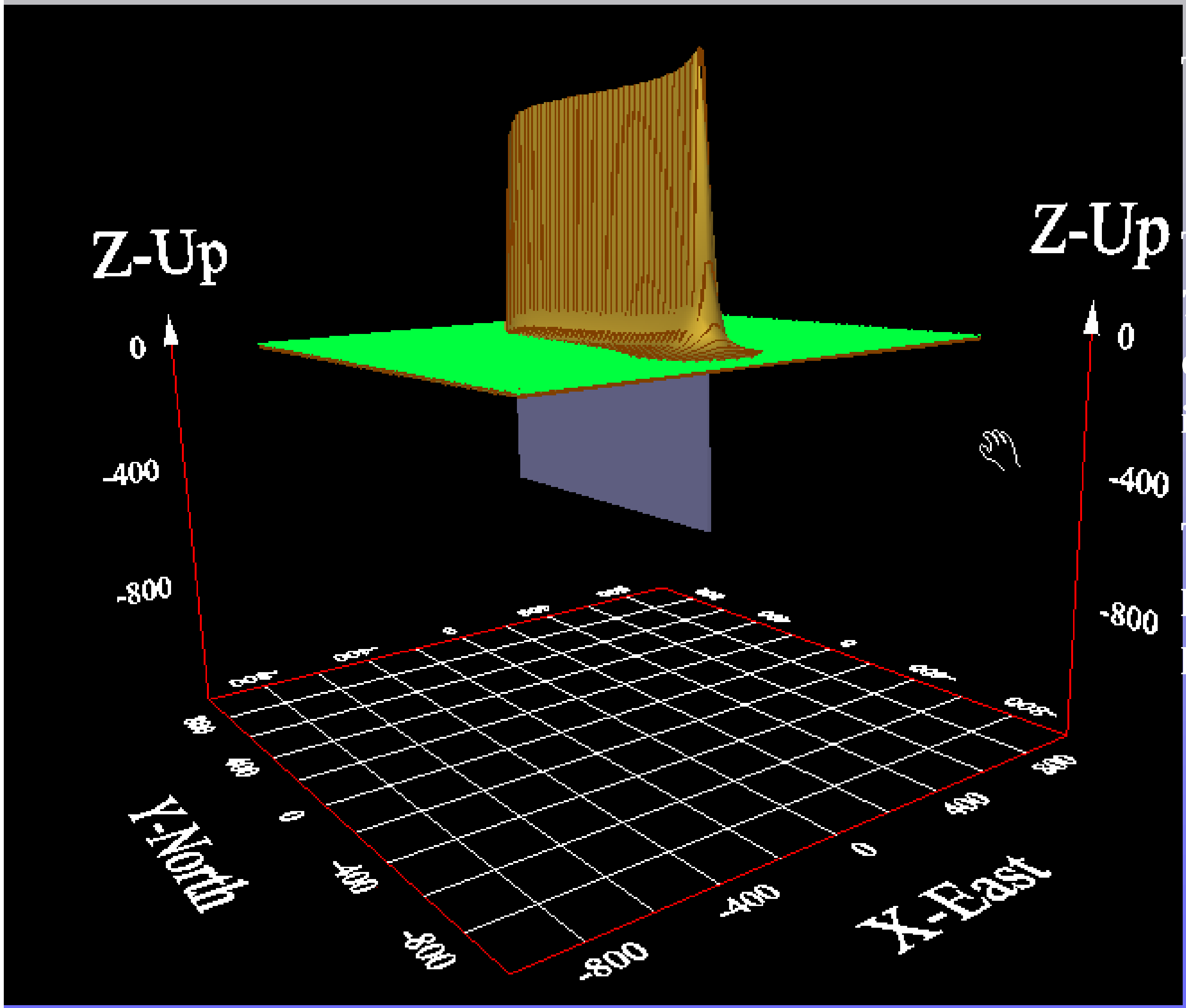
Fourier Transform Processing for Derivatives

1) All processed signals from the TMI data are generated by some transformation from the TMI derivatives (e.g. vertical derivatives, analytic signal, reduction-to-the pole). Traditionally, these derivatives are derived from Fourier transform of the TMI generally by FFT techniques.

***2) PetRos EiKon* has extended its simulation algorithms to synthesize the derivatives of the TMI. These derivatives are not calculated by difference techniques from data at different positions on the grid but rather by extending the quasi-analytic formulation to calculate instantaneous (by position) derivatives at each data point. This is done by extending the Integral Equation formulation for the components to spatial derivatives of the components. The extensions are available for the normal Born calculation (magnetization parallel to Earth's field) and for non-linear effects (e.g. magnetic channelling, de-magnetization, interacting structures, remanent magnetization, etc).**

Combining these two techniques in our newest software release allows the investigation of many aspects of traditional TMI processing. We will examine a couple of aspects here.

Fourier Transform Processing for Derivatives - The Model



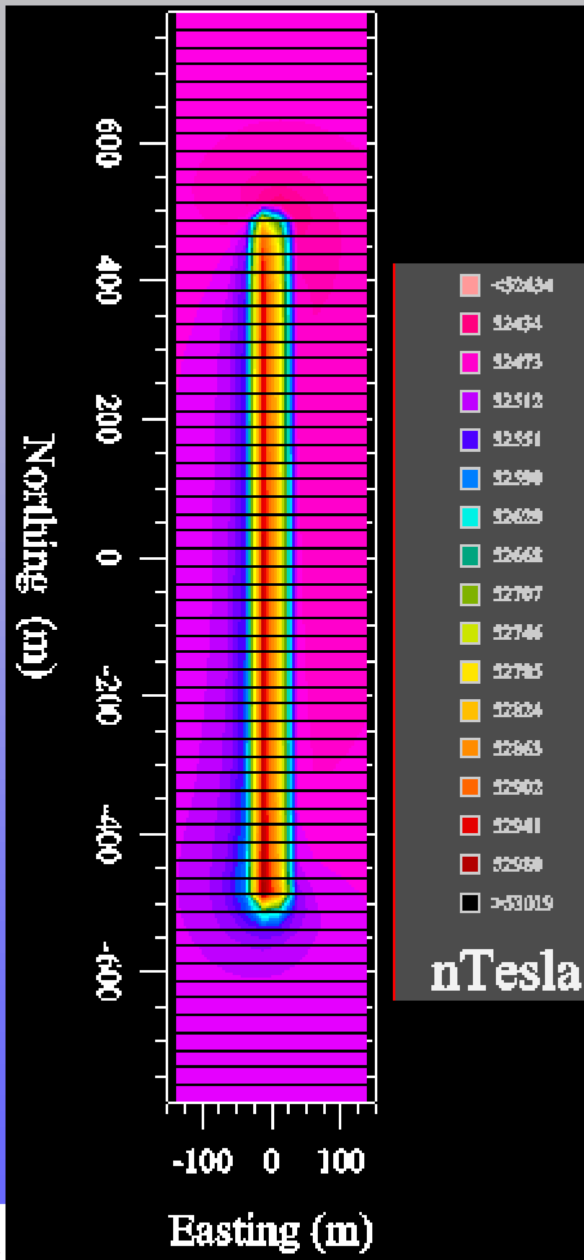
The model for this study is a thin dyke, 1km in length, striking N-S.

The inclination of the Earth's field is 75 degrees and the declination is 20 degrees East of North. The intensity is 52,500 nT.

The survey area is 1575 x 1575m, profile lines are 25m apart and data points are 25m apart.

For this example, the synthetic data is determined on a regular grid to illustrate varies features.

Fourier Transform Processing for Derivatives - TMI



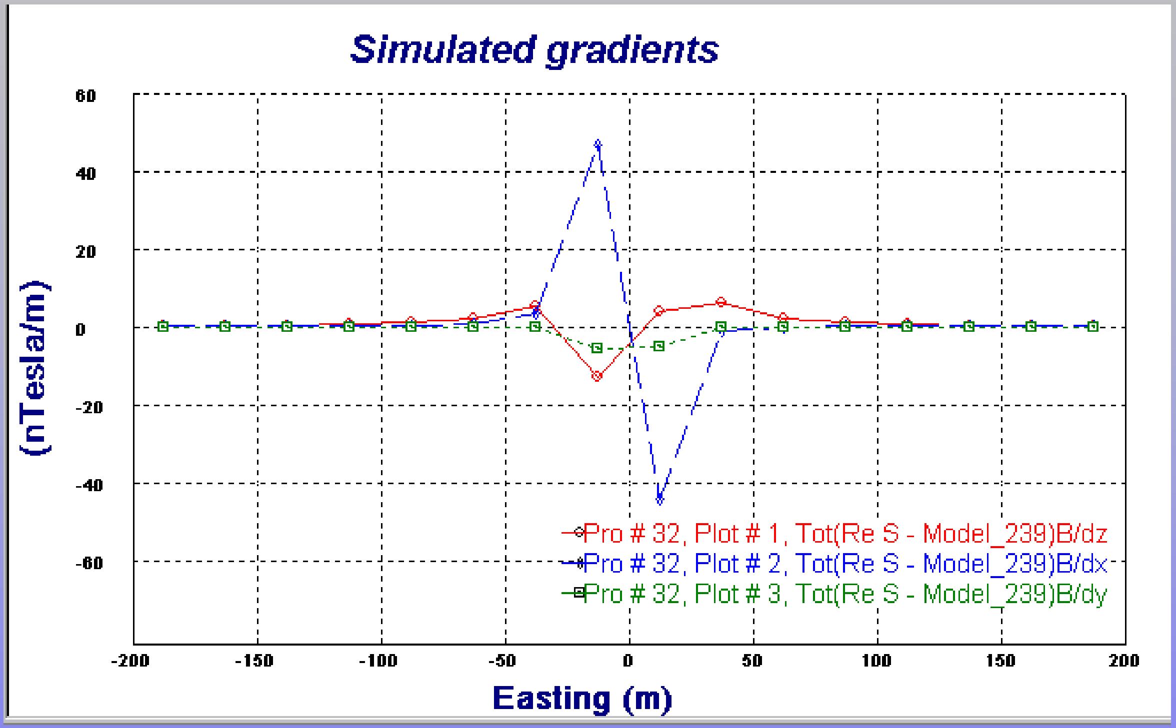
TMI Simulated

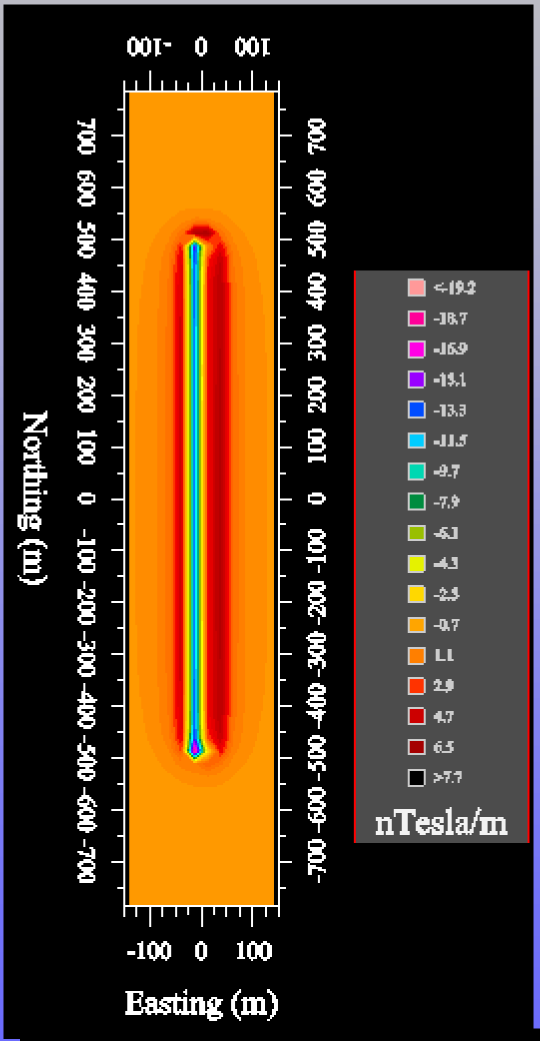
Here we display the simulated TMI. The interpolated data is on a grid which is exactly that of the simulated data points.

Note the asymmetry in the TMI response due to the inclination and declination of the earth's field.

The dyke comes to within 5m of the earth's surface and has a depth extent of 500m.

Fourier Transform Processing for Derivatives - Derivatives





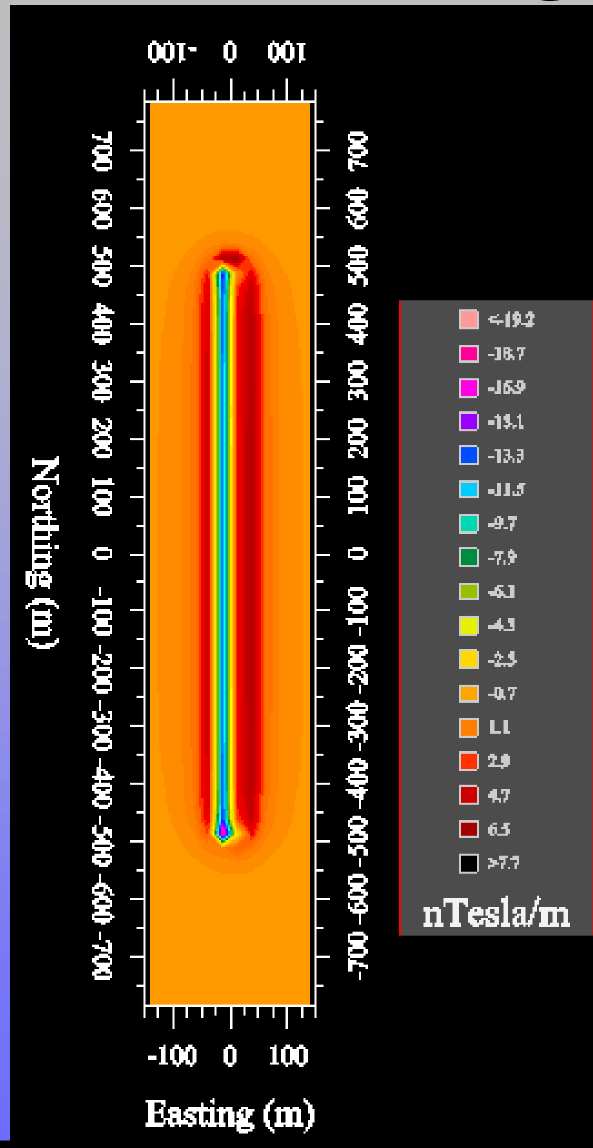
Simulated Vertical Gradients

Here we display the simulated derivates across a central line of the anomaly. And to the right, the contoured vertical gradients on the original data grid.

Fourier Transform Processing for Derivatives

- Derivatives

Vertical Gradient by FFT



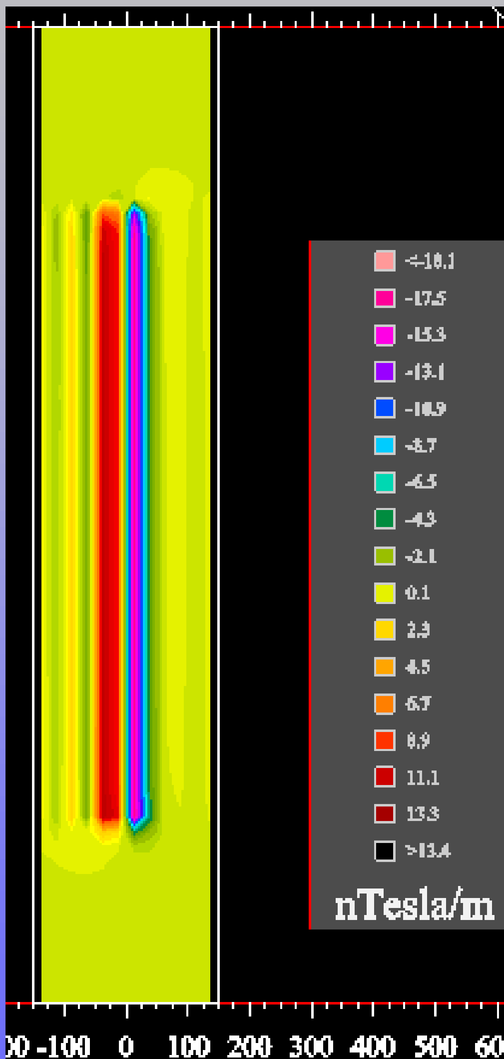
Vertical Gradient by Simulation

This is an extremely interesting Figure as it demonstrates that the Technique of deriving derivatives By FFT is more-or-less justified.

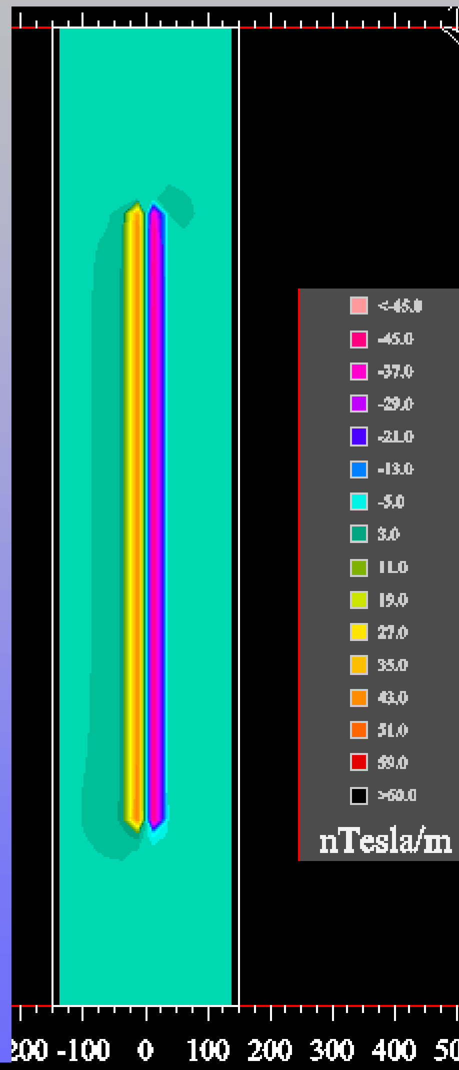
For those unfamiliar with the proof Of such techniques, the original Mathematical justification for the Fourier transforming for the Derivatives is not fully proven. Also, Using an FFT for the Fourier transform is somewhat contrary Is proper mathematics.

However, as can be seen by comparison of the figures. The FFT technique does overestimate the Vertical gradient and imparts variation which is also not actually in the derivative.

Fourier Transform Processing for Derivatives - Derivatives



In-Line Gradient by FFT



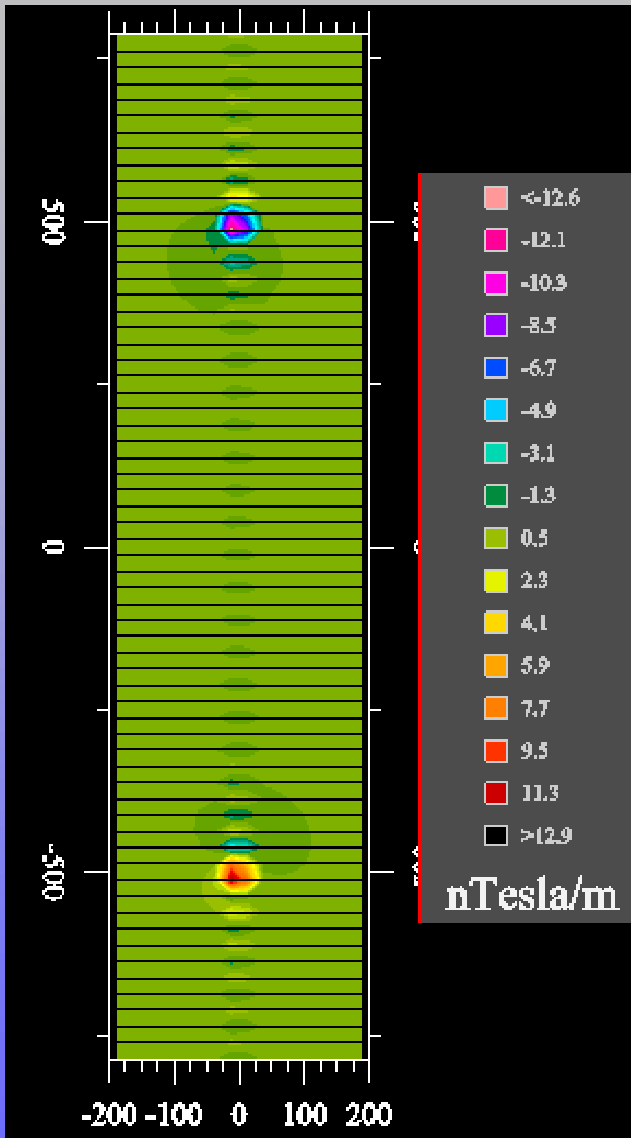
In-Line Gradient by Simulation

What is more interesting is the comparison of the in-line horizontal derivatives (d/dx).

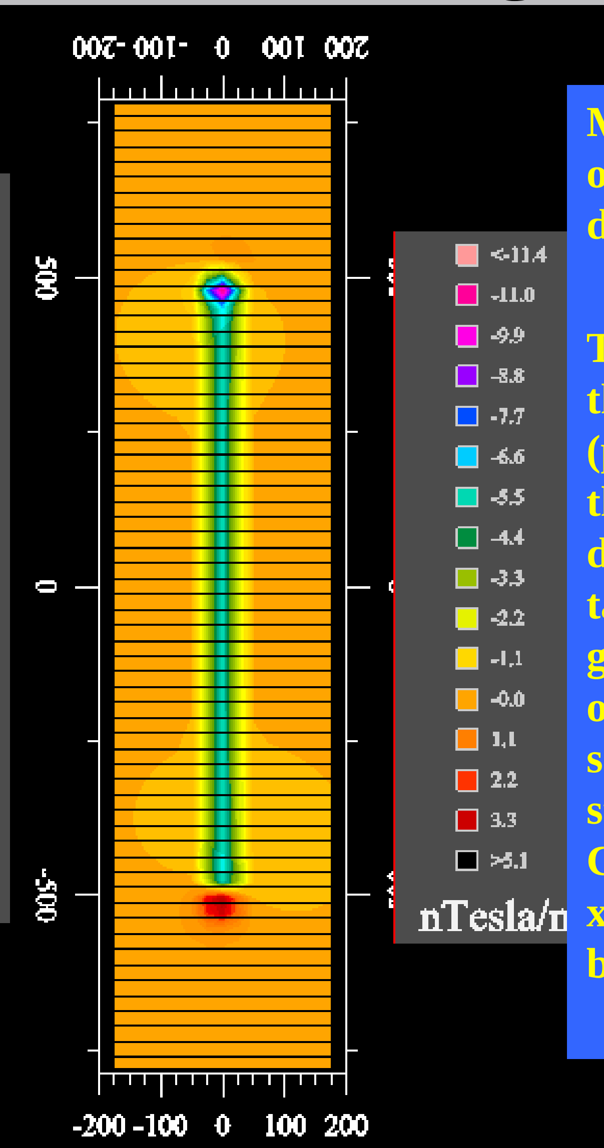
Note that the in-derivatives by FFT are significantly underestimate. Compare the figures and that of the x-y plot of this derivatives (2 pages back).

Note the rippling caused by the FFT as expected.

Fourier Transform Processing for Derivatives - Derivatives



Cross-Line Gradient by FFT



Cross-Line Gradient by Simulation

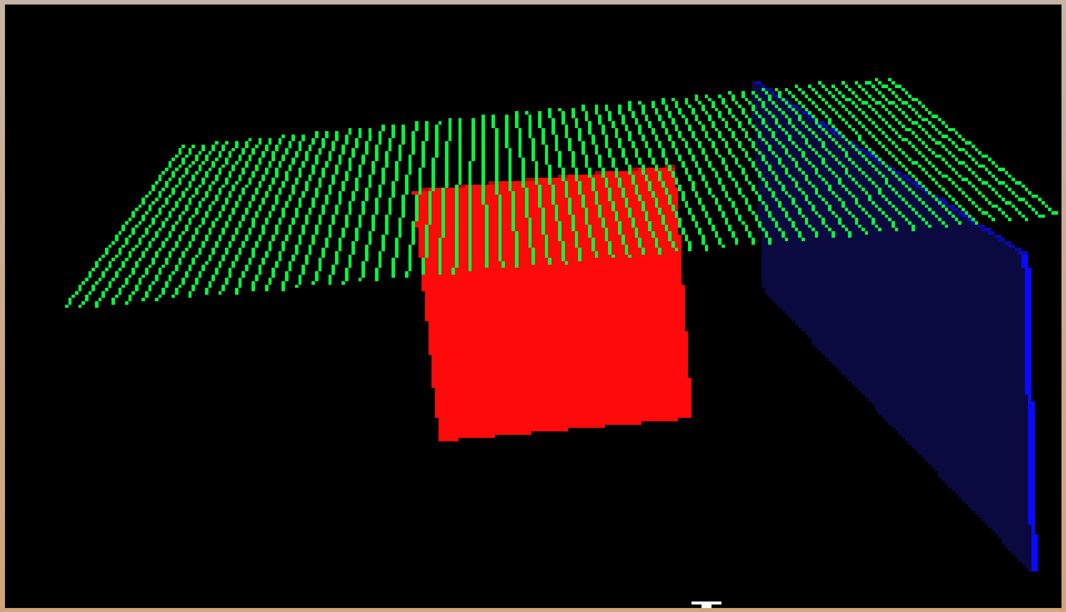
More disturbing is the comparison of the cross-line horizontal derivatives (d/dy).

The FFT technique overestimates the derivatives at the ends (particularly in the south). Shows the rippling effects caused by the discrete samples in the FFT and the taper window at the edge of the grid. And cannot discriminate the object in the center but rather shows the object as 2 dipole like structures only at the ends. Compare the figures and that of the x-y plot of this derivatives (3 pages back).

Gridding (Interpolation) with Derivative Information

Example 3

Another interesting aspect of measuring gradients is the consideration that the derivatives may be used to enhance (increase resolution) of the data grids. This would be accomplished by using interpolation techniques which would utilize the measured gradients to increase the density of the interpolated output grid from the profile data.



We can also investigate this aspects with the use of our new software tools.

Here we will consider a new model beneath the previous data grid. The fault is reduced in size and another object (larger than the grid) is introduced which is sub-parallel to the data lines (i.e. almost parallel).

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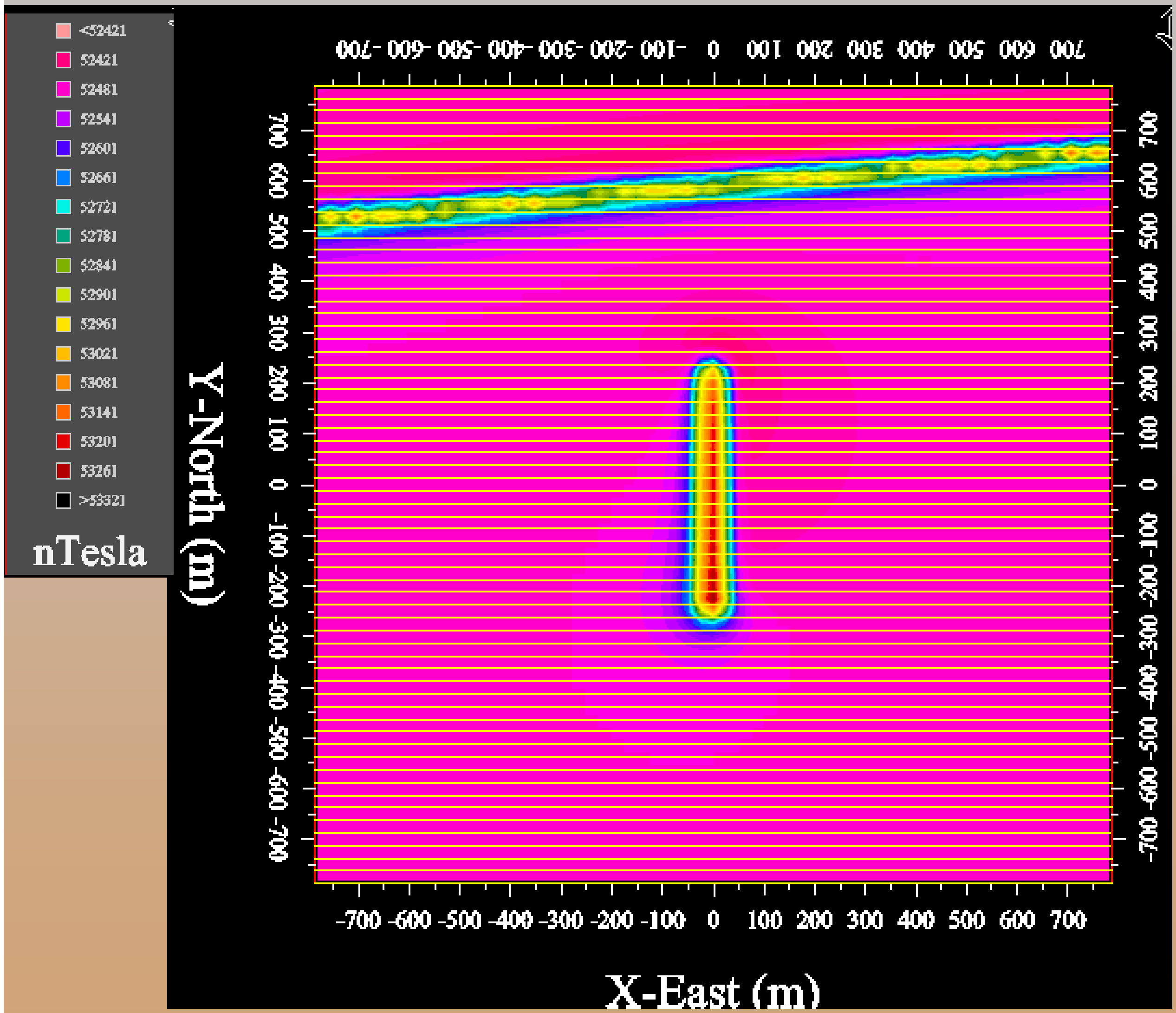
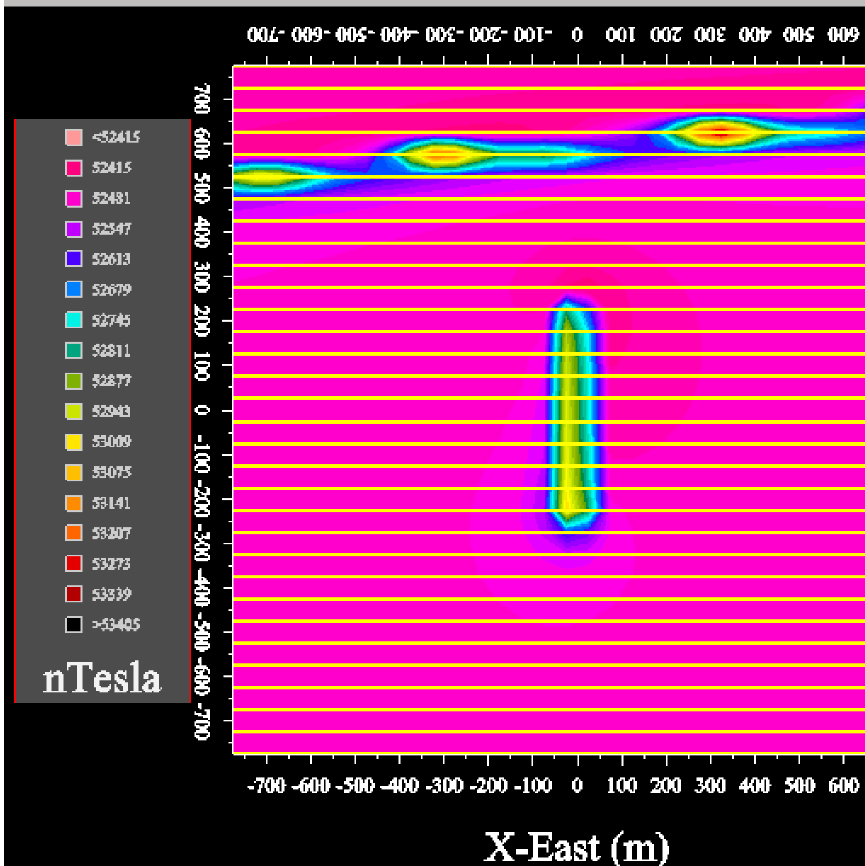
Gridding (Interpolation) with Derivative Information

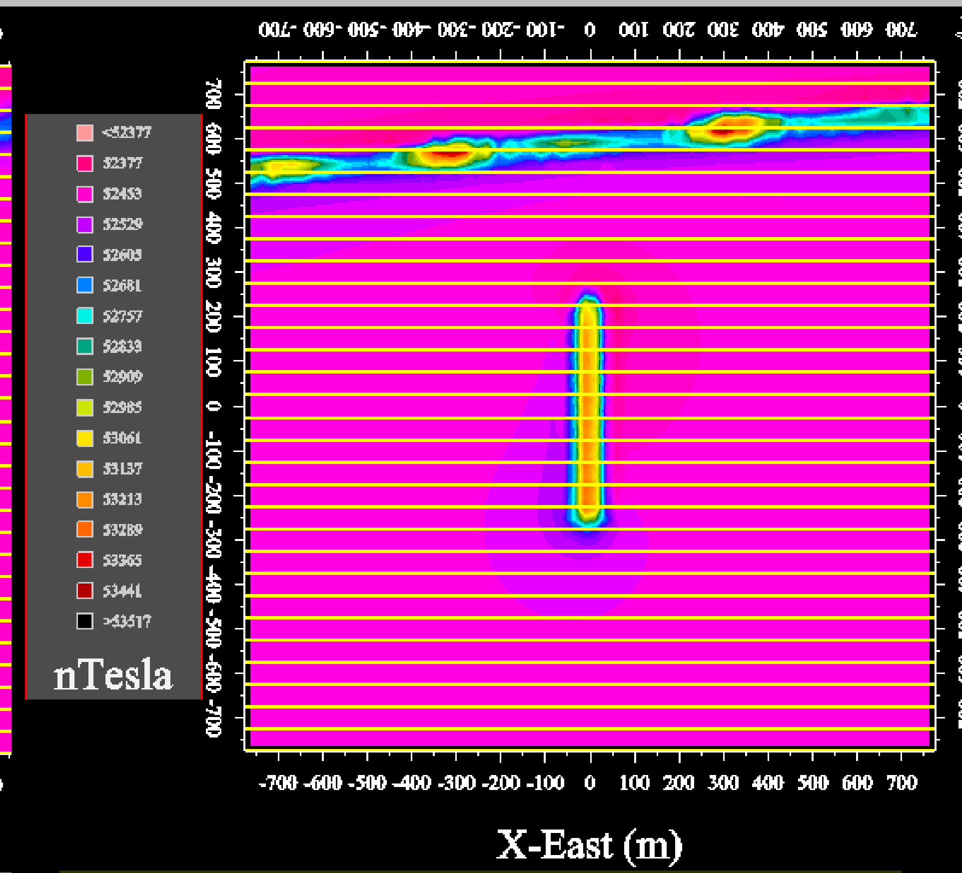


For this grid, both objects are clearly delineated by the 25x25m data sampling.

Gridding (Interpolation) with Derivative Information



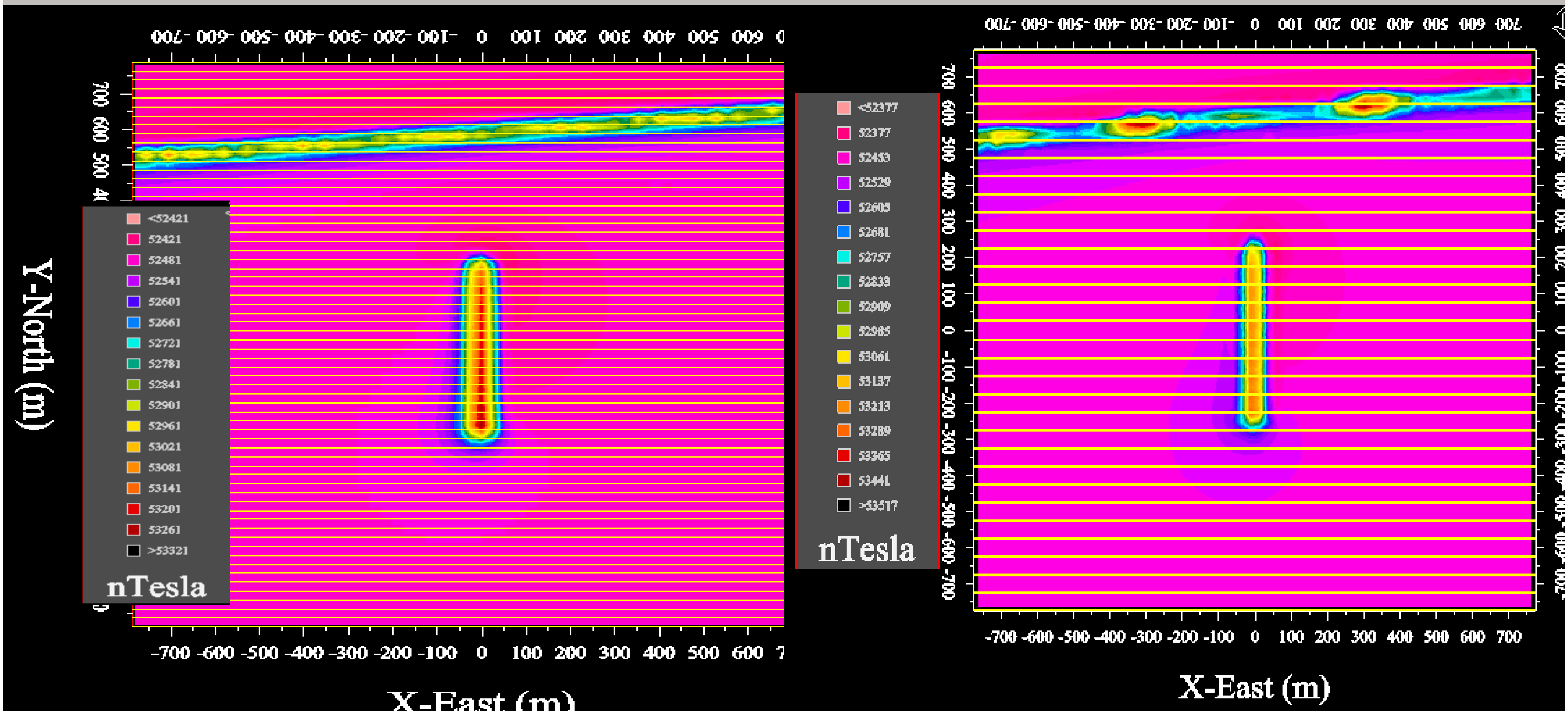
TMI from 50x50m grid



TMI from 50x50m grid using gradients

The use of the gradients does improve the resolution both of the E-W structure but almost more clearly outlines the N-S structure. Data sampling in these grids are not preciously as in the more dense grid (previous figure). However, the gradient gridding very closely reproduces the high density "data".

Gridding (Interpolation) with Derivative Information



TMI from 25x25m grid

TMI from 50x50m grid using gradients

The use of the gradients does improve the resolution both of the E-W structure but almost more clearly outlines the N-S structure. Data sampling in these grids are not preciously as in the more dense grid (previous figure). However, the gradient gridding very closely reproduces the high density "data".

3 Examples of *PetRos EiKon* research as pertains to the use of measured TMI derivatives

Conclusions:

3 pertinent examples of the use of magnetic gradients showing different aspects of our research project.

R.W. Groom, PhD