3D MODELLING – APPLICATION OF THE LOCALIZED NON-LINEAR APPROXIMATOR TO NEAR SURFACE APPLICATIONS

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- Localized Non-Linear Approximator (LN) is an "operator" technique for simulation

- initial development (1991) for cross well EM for reservoir characterization

- extended and utilized successfully for a range of mining applications (1992 – 1997)

- offers potential for near-surface applications

Basic Equations for EM

the complex electrical contrast (conductivity plus permittivity)

$$Q(r) = i\omega\mu_0[\sigma(r) - \sigma_b] + \omega^2\mu_0[\varepsilon(r) - \varepsilon_b]$$

where σ can be dispersive (e.g. Cole-Cole parameters)

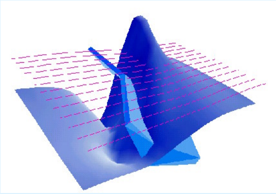
the background or incident field is given by

$$\nabla\times\nabla\times E_b - k_b^2 E_b = i\omega\mu_0 J_s + \nabla\times M_s$$

generally the background is defined as a layered environment

<u>Electric field integral solution for a single scatterer (object)</u>

$$\boxed{E(r) = E_b(r) + \int dr'\,\overline{\overline{G}}(r,r')\bullet Q(r')E(r')}$$



analogous eqn for magnetics

LN Solution for EM

For an internal point we assume the electric field is locally constant

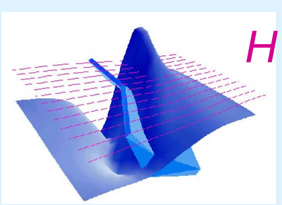
$$E(r) \approx E_b(r) + \int dr' \,\overline{\overline{G(r,r')}} \bullet Q(r')E(r)$$

Which leads to an approximate solution for the internal field

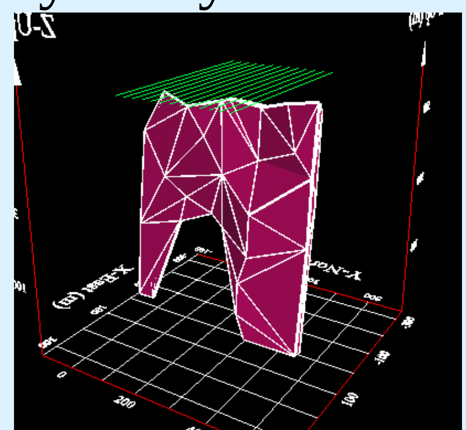
$$E(r) = \overline{\overline{\Gamma}}(r)E_b(r) = \left[\overline{\overline{I}} - Q\overline{\overline{L}}\right]^{-1} E_b(r), \quad \overline{\overline{L}} = \int_{V^{(i)}} dr' \overline{\overline{G}}$$

The scattering operator, Γ, is determined quasi-analytically

E,H are determined by integrating the
(scattering currents x
their propagation operators)



$$H(r) = H_b(r) + \frac{\Lambda}{i\mu\omega} \times \int dr' G(r,r') \cdot Q(r')\,\Gamma(r',\omega)E_b(r')$$



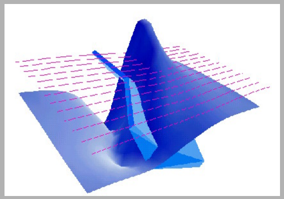
LN Advantages and Disadvantages

- **speed** - O (N)
  many, many times faster than matrix solution techniques

- **small memory allocation** - O (N)
  in 1994, 3D simulation on a 286 with 4M RAM

- **“accuracy”** for some aspects
  e.g excellent accuracy in EM for galvanic excitation

- **flexibility**
  virtual unlimited model complexity, simple grid definitions

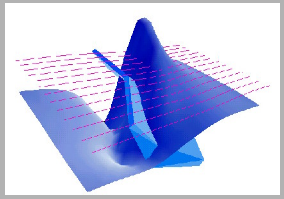


LN Advantages and Disadvantages

- **single-body problem**
- **inaccuracy for some types of physical responses**
  Inductive modes (magnetic field excitation)
- **shape capabilities**
- **internal gradients in electrical/magnetic properties**

Localized Nonlinear (LN) Approximation Extensions

- ✓ Inductive modes
- ✓ Multiple body problems
- ✓ Magnetic effects, static and time-varying
- ✓ Polyhedral primitives



Localized Nonlinear (LN) Approximation Extensions

✓ Inductive modes

- By design, LN accurate only when the local gradients of the internal field are insignificant

  e.g. strong inductive coupling of source and scatterer

- Inductive Non-linear (ILN) Approximator incorporates the field gradients

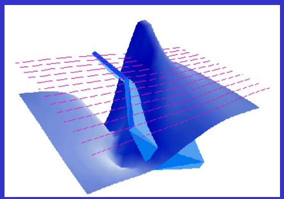
  12x12 local system projecting the incident electric field

  and its gradients into the internal field

$$\nabla^* E = \nabla^* E_b + Q\, \nabla^* \int dr' \,\overline{\overline{G}} \bullet [E(r) + \nabla E(r) \bullet (r' - r)]$$

where

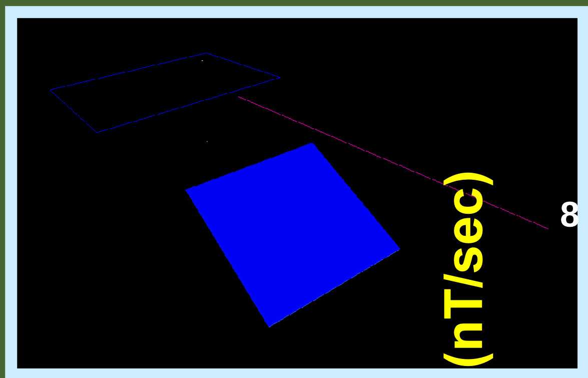
$$\nabla^* = [1, \nabla\,] = [1, \partial/\partial x, \partial/\partial y, \partial/\partial z\,]$$

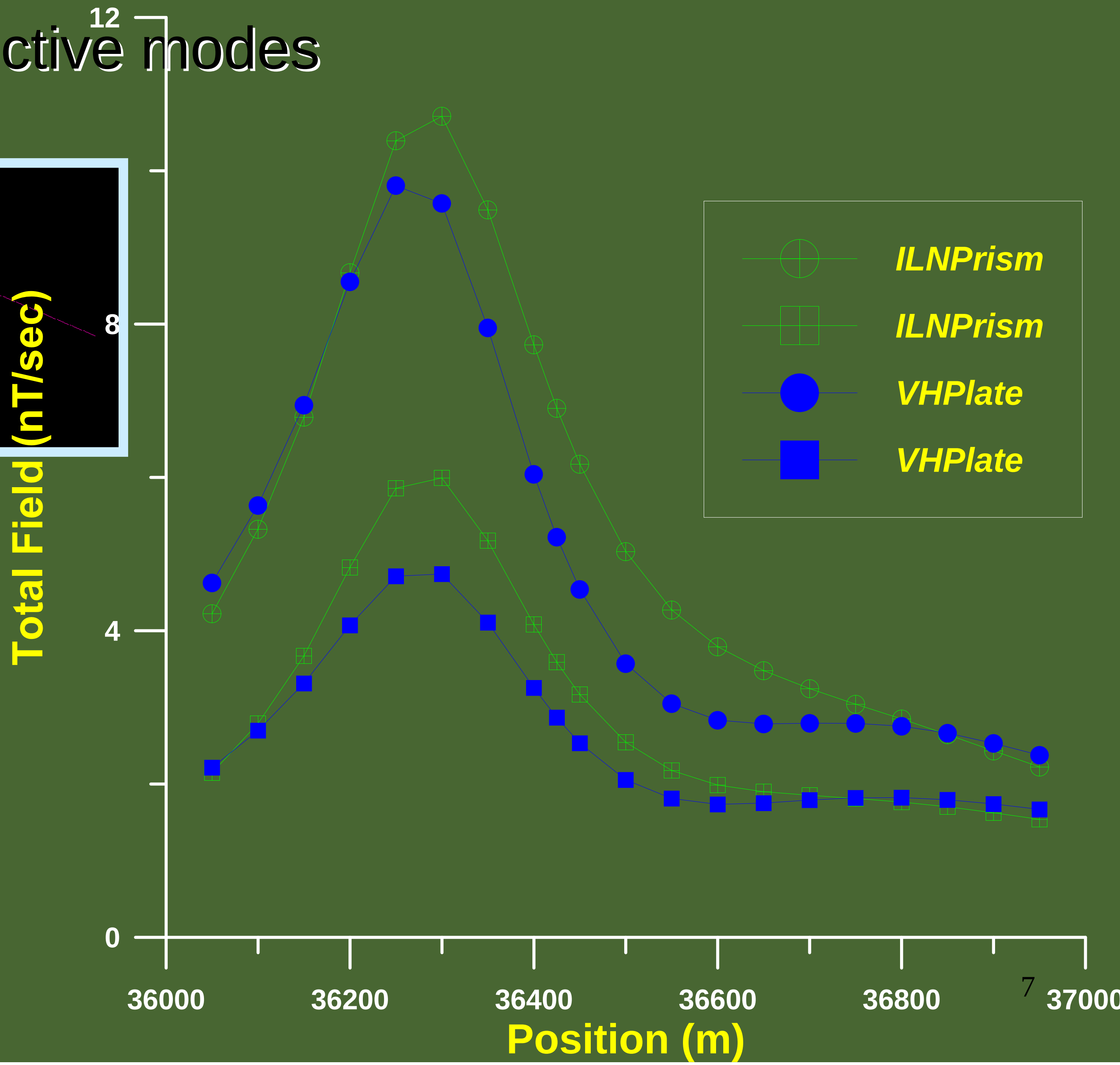


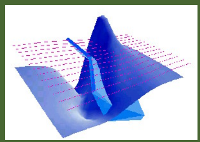
<u>Field and Gradients separated numerically</u>

Localized Nonlinear (LN) Approximation Extensions

✓ Inductive modes

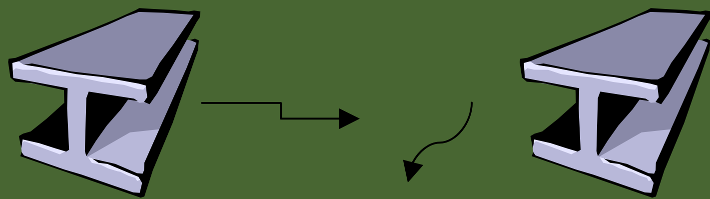




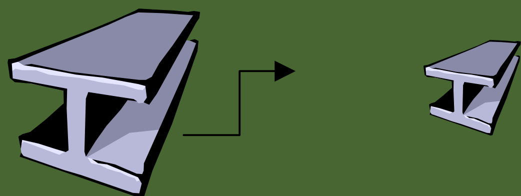


Localized Nonlinear (LN) Approximation Extensions

✓ Multiple Bodies

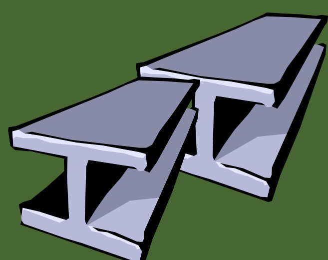


Superposition



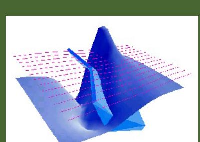
"Far Field Scattering"

$$E_{s,FF} = \sum_{i=1}^{n} \int_{V^{(i)}} dr_{i'} \,\overline{\overline{G}}(r, r_{i'}) \bullet \overline{\overline{\Gamma}}^{(i)} \bullet \left[E_b^{(i)} + \sum_{j \neq i}^{n} \int_{V^{(j)}} dr_{j'} \,\overline{\overline{G}}(r_{i'}, r_{j'}) \bullet \overline{\overline{\Gamma}}^{(j)} E_b^{(j)} \right]$$



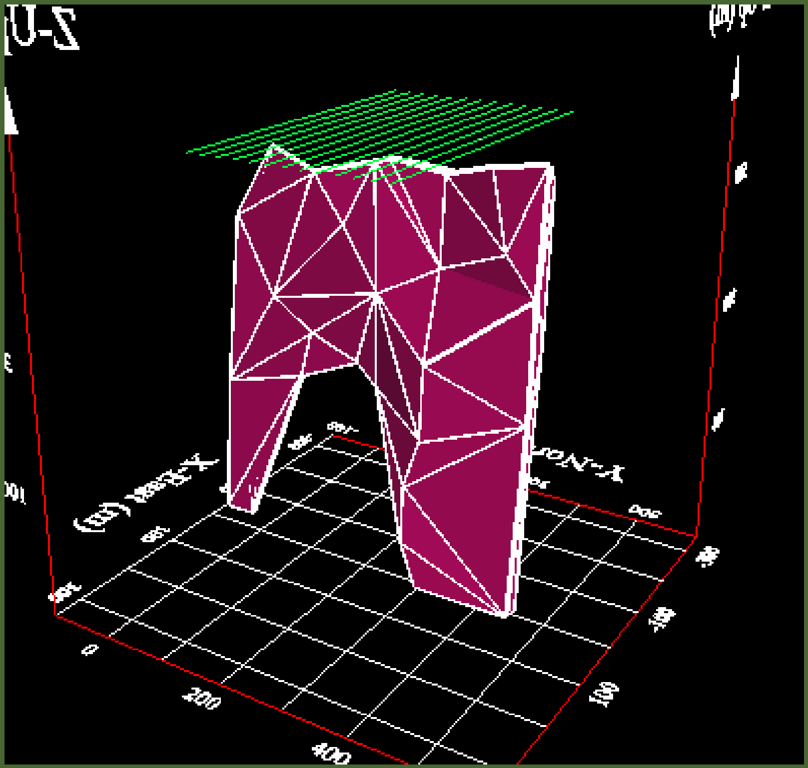
Near-Field

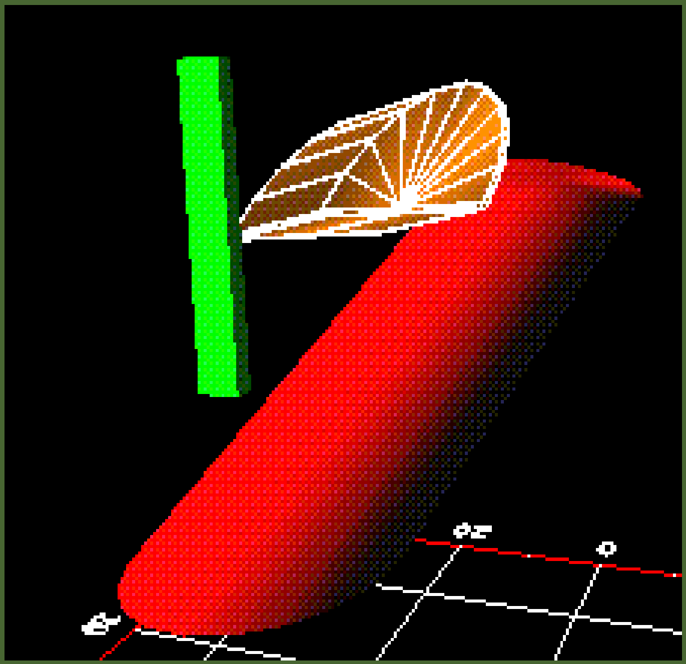
electrical or magnetic current flow between objects



Localized Nonlinear (LN) Approximation Extensions

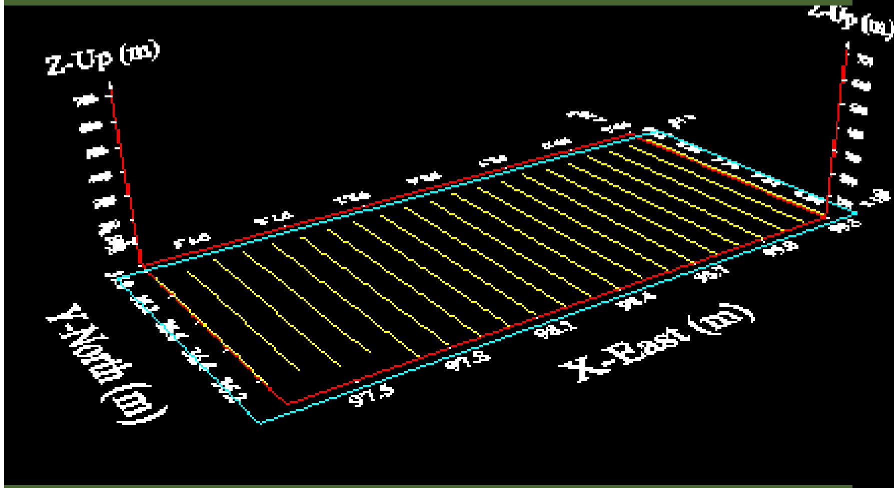
- ✓ Polyhedras

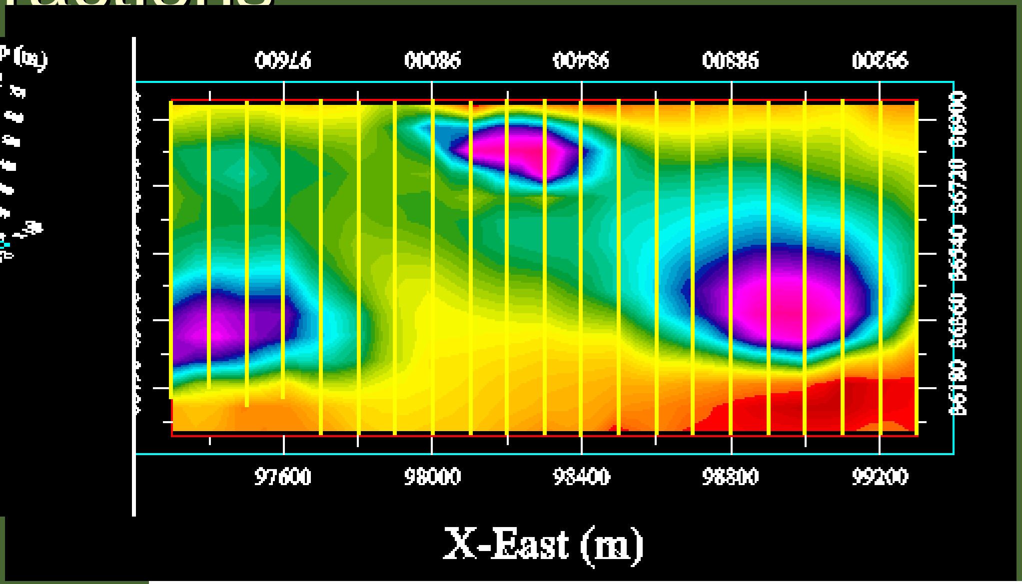




Localized Nonlinear (LN) Approximation Extensions

✓ Polyhedras + Interactions

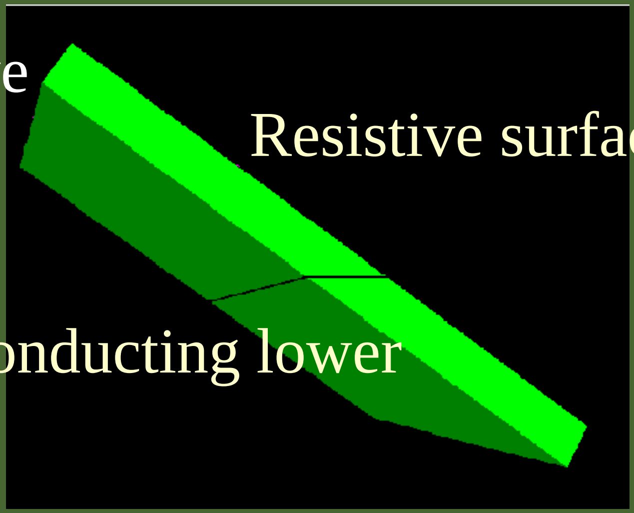


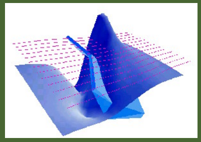


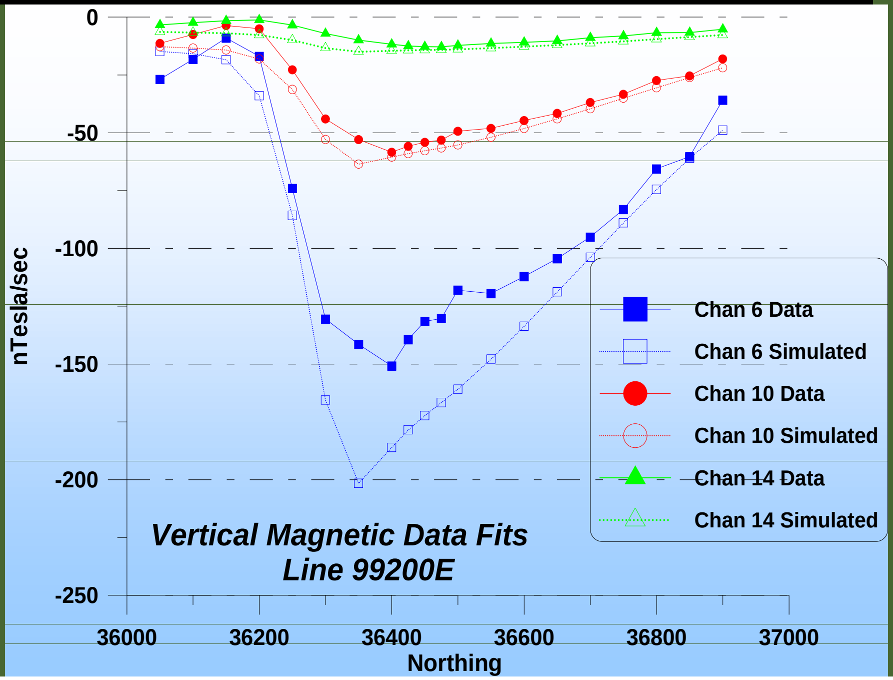
Intrusive

Resistive surface

Conducting lower







Localized Nonlinear (LN) Approximation Extensions

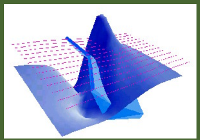
✓ Magnetics and Magnetic Effects

- **magnetic effects in IP and cross hole RIM**

- **inductive effects of susceptibility in TEM and FEM**

-DC magnetics – Full 3D capabilities including

- **strong magnetization effects**
- **magnetic channeling**
- **magnetic interactions**



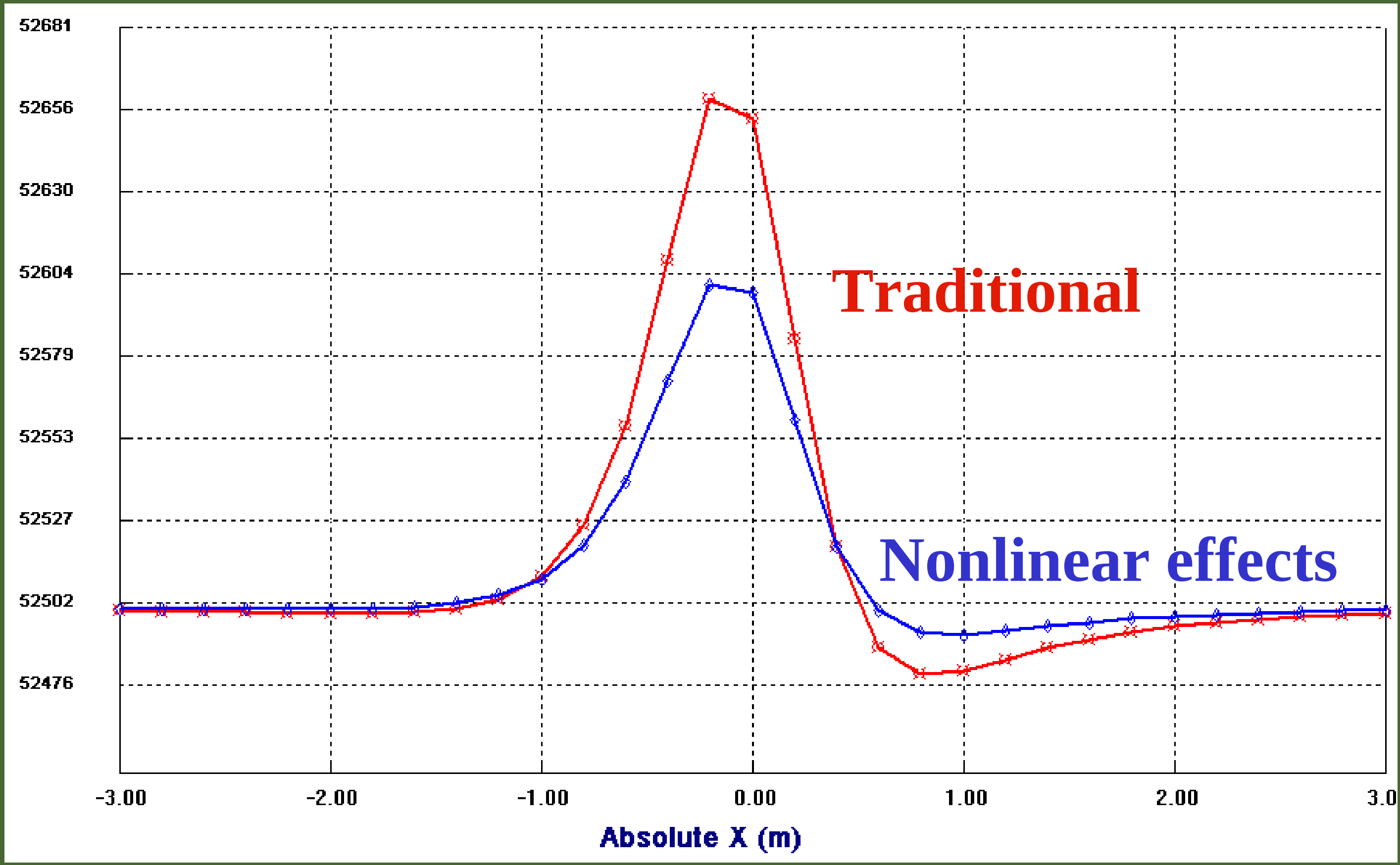
Localized Nonlinear (LN) Approximation Extensions

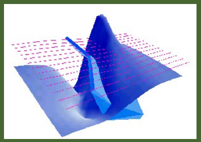
✓ Magnetics and Magnetic Effects

-DC magnetics – Full 3D capabilities including

- ➢ strong magnetization effects

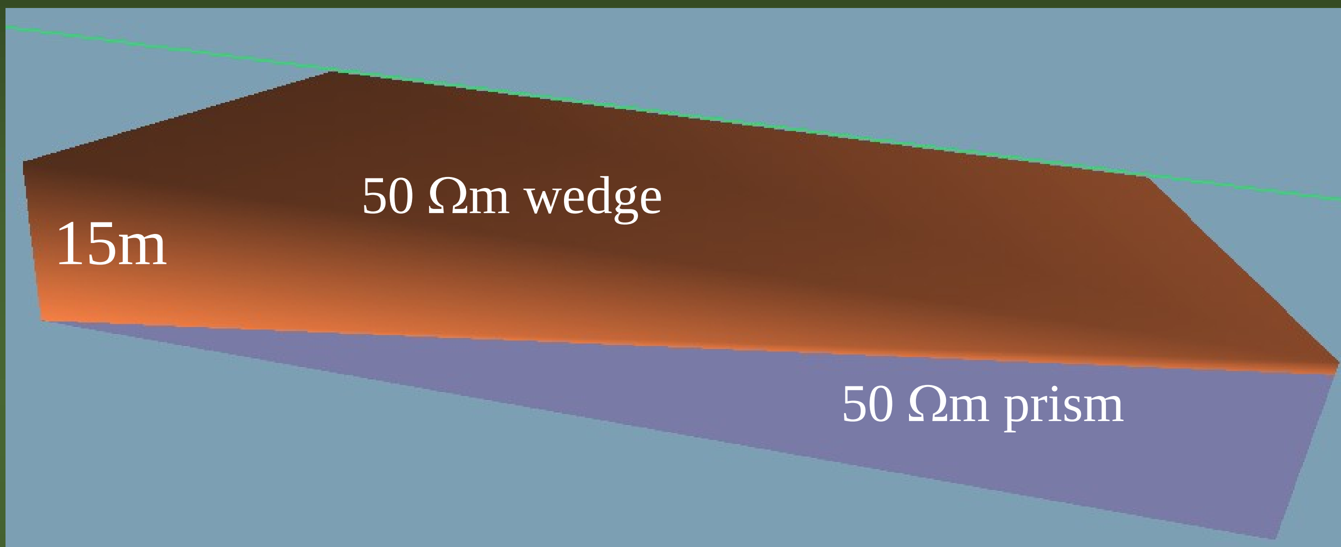
300mm
shell model





➢+ remanent + induced-remanent interactions

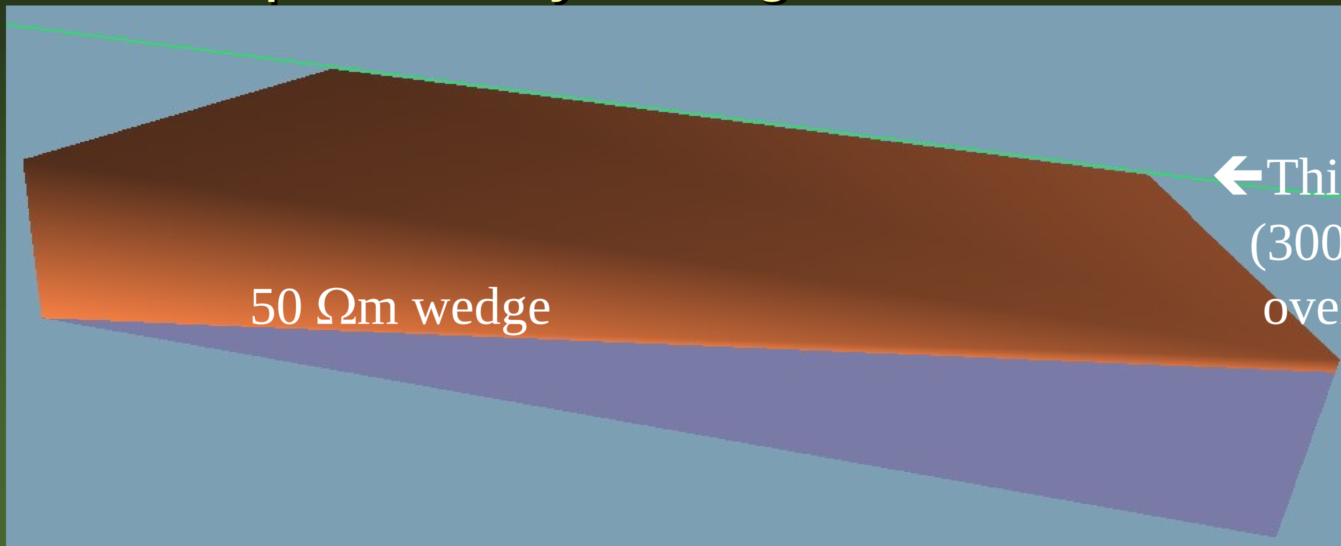
FEM Example – Clay Wedge



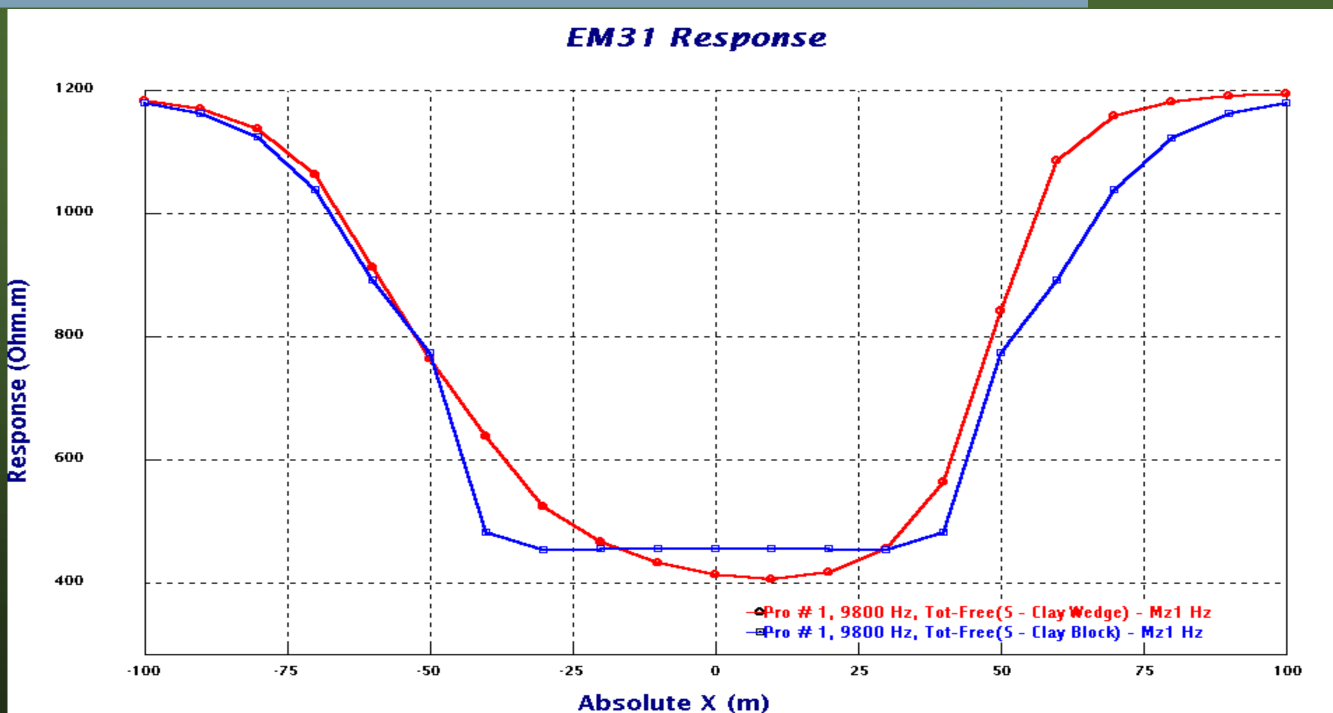
🡐Thin resistive cover (3000 Ωm) over 400 Ωm

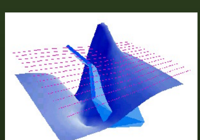
Clay Wedge Model - EM31

FEM Example – Clay Wedge



← Thin resistive cover (3000 Ωm) over 400 Ωm





Conclusions

Non-Linear Scattering Operator Technique

Versatile Technique allowing:

- ❖Easy use and development
- ❖Range of physical simulation abilities
- ❖Large model calculation capability
- ❖EM, Magnetics, Resistivity, IP
- ❖Speed

Research Direction

- ❖EM: susceptibility effects, improved induction
- ❖Resistivity: Non-Born 3D Resistivity Inversion
- ❖Magnetics: UXO applications
  - Remanent – Induced Interaction
  - Non-Born (weak) Inversion