# 1D FEM Inversion

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$$d = F(m)$$

$d \rightarrow$ ***data vector of dimension N***

$m \rightarrow$ **model vector of dimension M**

F - physical relationship describing the data as a function of the earth model - *In practice an approximation*

## Approach

- Optimized Inversion - modern practical approach

# Optimization

General concept is to start with an initial guess and go looking for the best fitting model by minimizing a given function using an iteration process.

**Critical factors to Optimization Results:**

**1. Accurate Forward Simulation Algorithm** - the engine that drives the inversion

Layered Earth modelling code which computes for (2) parameters: resistivity and susceptibility in every layer. The theory is based on Maxwell's equations, Christiensen and Anderson's work and can be found in any standard geophysics textbook. However, we have numerically improved the technique and made it our own.

**2. OCCAM Type Inversion** (L2 + OCCAM)

Smooth variation in model parameters. Original work was done for 1D ad 2D MT and then 3D Magnetic Data - work pioneered by Steve Constable and John Booker among many others. We borrowed the technique from our 3D inversion and applied it to layered earth. Because it is smooth, the search does not move very far and you need a constraint to push the search in the opposite direction so that it does not get stuck in a rut and thus the application of Conjugate Gradient techniques.

$$\phi(m) = \lambda\, \phi_d(m) + \phi_m(m)$$

$\phi(m)$ - functional to be minimized

$\phi_d(m)$ - data misfit function

$\phi_m(m)$ - model misfit function

$\lambda$ - Lagrangian multiplier - regularization weight

## Occam style model misfit function

$$\boldsymbol{\phi_m(m) = \alpha_0 \int (z)\, [\, m(r) - m^0(r)\,]^2\, dv}$$

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**3. Good Minimization Technique** - Conjugate Gradient Constraint

Search for the best model under some kind of constraint. We want to minimize the difference between the data and the estimated data. Conjugate Gradient is an efficient way of doing so. More specifically, Conjugate Gradient is a technique to define the search direction that moves the iteration from one point to another. Uses the derivative information to construct two sequences of orthogonal vectors to define the search direction at a given iteration. Then by trial and error (line search) to move to the local minimum in that direction. The iteration stops when the gradient has achieved the required minimum value. This is an unconstrained minimization technique where the bounds on the parameters are imposed after the search is completed.

We have added the option to join layers of similar resistvities at the end to form discrete layers and then test the fit. If it does not "fit", then the program will break them up again.

**4. Good Starting Model** - User defines the number of layers and starting resistivity and susceptibilitly above a basement

**5. Good Data of course a requirement**