Inversion of Ground Gravity and Airborne Gradient Data

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Society of Exploration Geophysicists, Houston, 2005

TOPICS

- **Interaction between Euler Deconvolution Technique and 3D inversion**
- **Equivalence source technique vs FFT**
  - *e.g. derivatives, continuation*
- **Conclusion**

Euler Deconvolution Technique-Review

Euler's Equation:

$$x_0 \frac{\partial G(x_i, y_i, z_i)}{\partial x} + y_0 \frac{\partial G(x_i, y_i, z_i)}{\partial y} + z_0 \frac{\partial G(x_i, y_i, z_i)}{\partial z} + NG_0 =$$

$$= NG(x_i, y_i, z_i) + x_i \frac{\partial G(x_i, y_i, z_i)}{\partial x} + y_i \frac{\partial G(x_i, y_i, z_i)}{\partial y} + z_i \frac{\partial G(x_i, y_i, z_i)}{\partial z}$$

$G(x_i, y_i, z_i)$ -measured data at locations, $i = 1, 2, \ldots, M$

G_0 - constant background field

N – structural index (fall-off rate of the field G)

(x_0, y_0, z_0) - anomalous source

Gravity Structural Indices

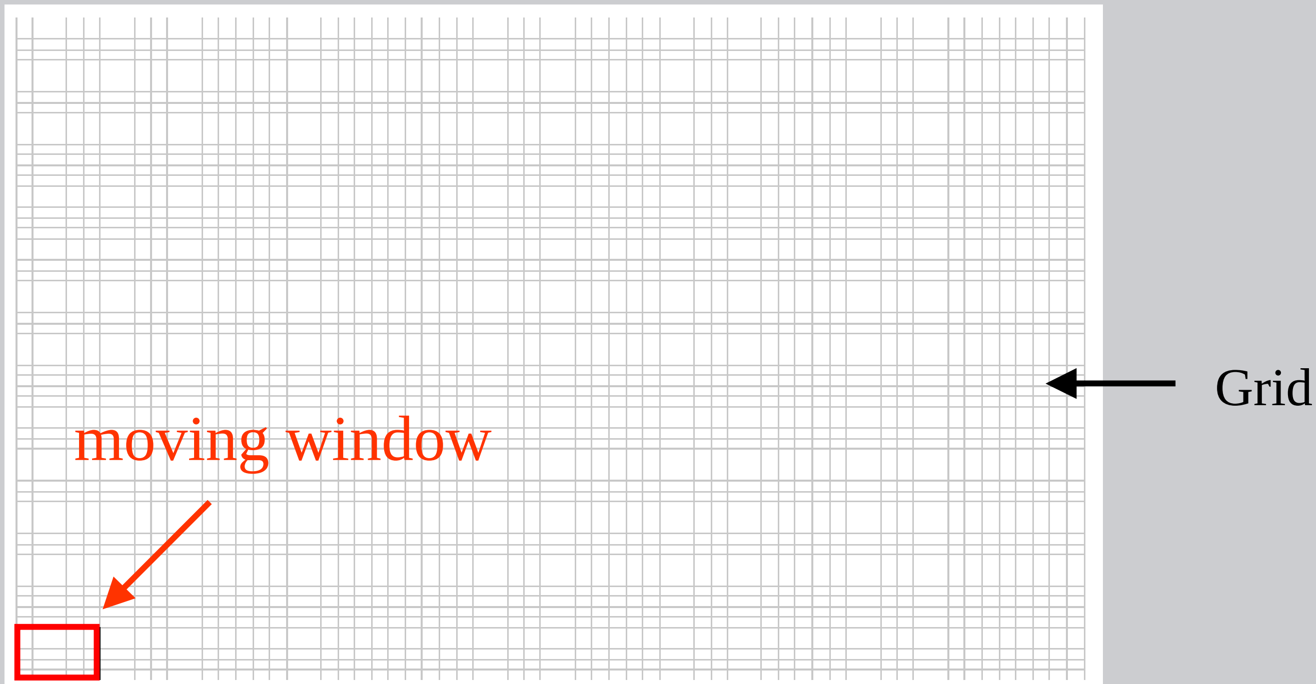
0 - sill/dyke/step

0.5 - ribbon

1 - pipe

2 - sphere

Euler Deconvolution – solutions and post-processing



3D-Euler deconvolution technique (Reid et al. (1990), Zhang et al, 2000)

Post-processing technique (Mikehailov et al. 2003)

Forward Modeling-gravity and gradients

Quasi-Analytic Solution:

- Polyhedron of triangular facets
- Integrals by M. Okabe (1979).

$$G_z(\vec{r}_0) == \gamma \iiint_V \rho(\vec{r}) \frac{z - z_0}{|\vec{r} - \vec{r}_0|^3} dv$$

$$G_{zx}(\vec{r}_0) == 3\gamma \iiint_V \rho(\vec{r}) \frac{(x - x_0)(z - z_0)}{|\vec{r} - \vec{r}_0|^5} dv$$

$$G_{zy}(\vec{r}_0) == 3\gamma \iiint_V \rho(\vec{r}) \frac{(y - y_0)(z - z_0)}{|\vec{r} - \vec{r}_0|^5} dv$$

$$G_{zz}(\vec{r}_0) = \gamma \iiint_V \rho(\vec{r}) \left[\frac{3(z - z_0)^2}{|\vec{r} - \vec{r}_0|^5} - \frac{1}{|\vec{r} - \vec{r}_0|^3} \right] dv$$

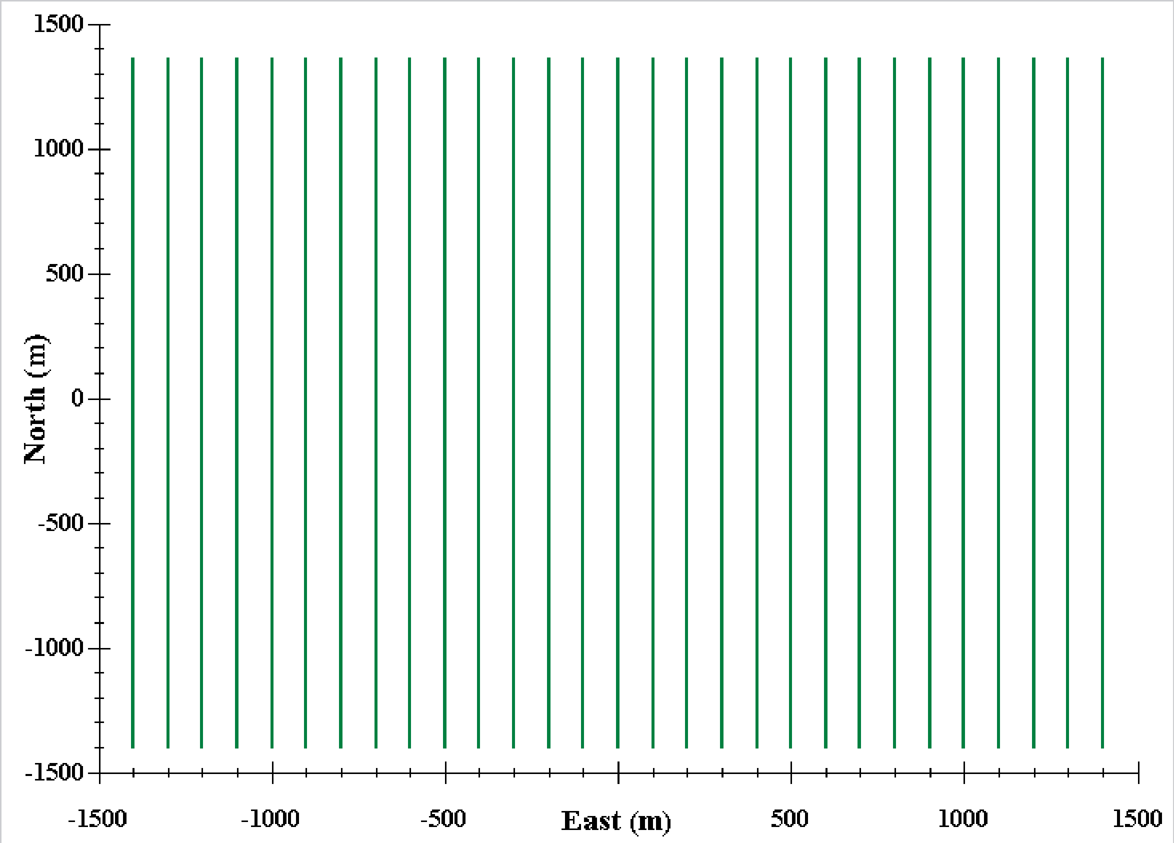
$\vec{r}_0 = (x_0, y_0, z_0)$ - observation location

$\vec{r} = (x, y, z)$ - source location

V - volume of the anomalous mass

γ - the gravitational constant

Synthetic Ground Gravity Survey

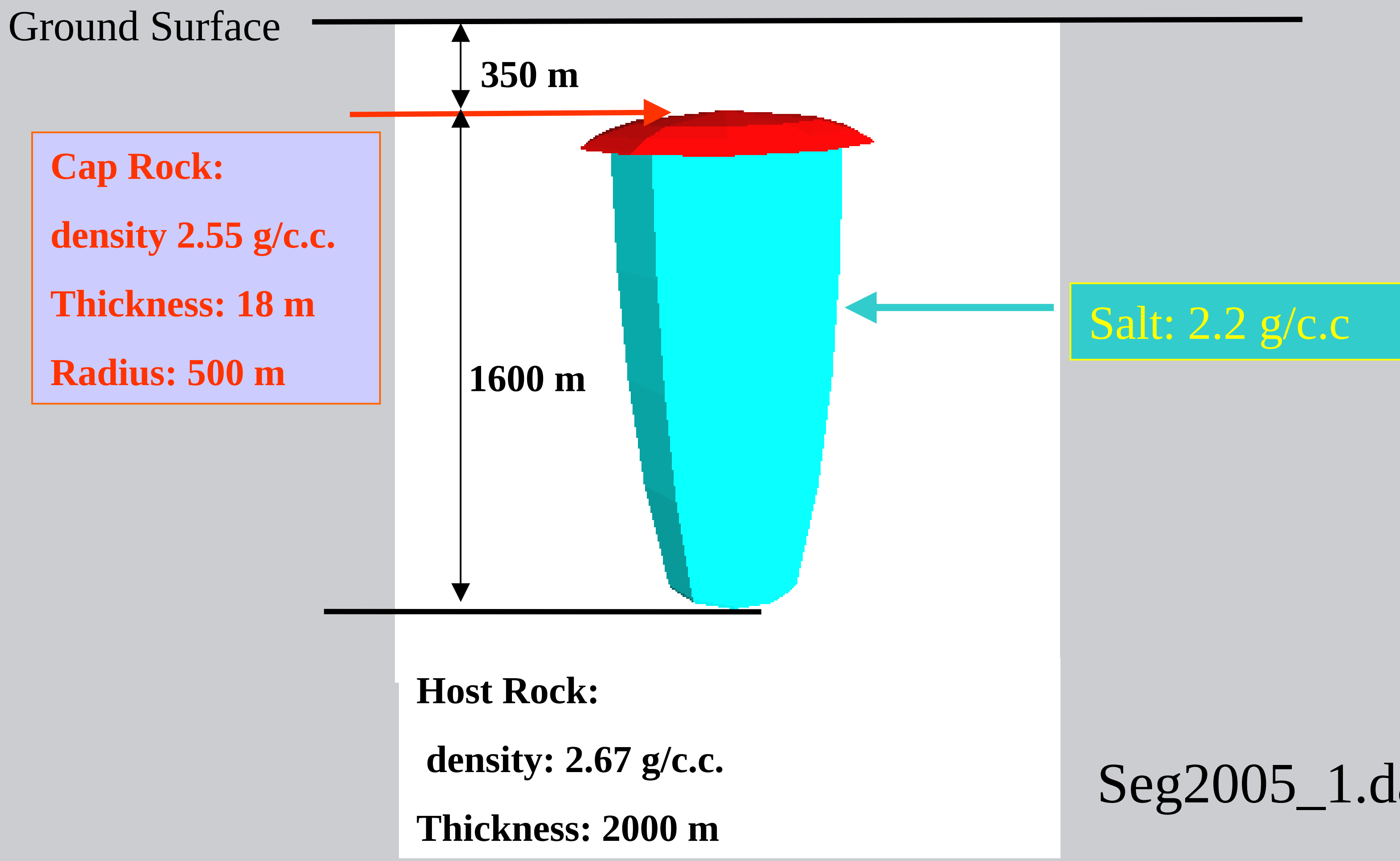


Seg2005_1.dat

- **30 lines of length 2800m along the NS direction.**
- **Inline sampling 40m. The distance across lines is 100m.**
- **The station elevation is 1 m from ground surface.**

Salt Dome Model

Based on Peters and Dugan (1945)

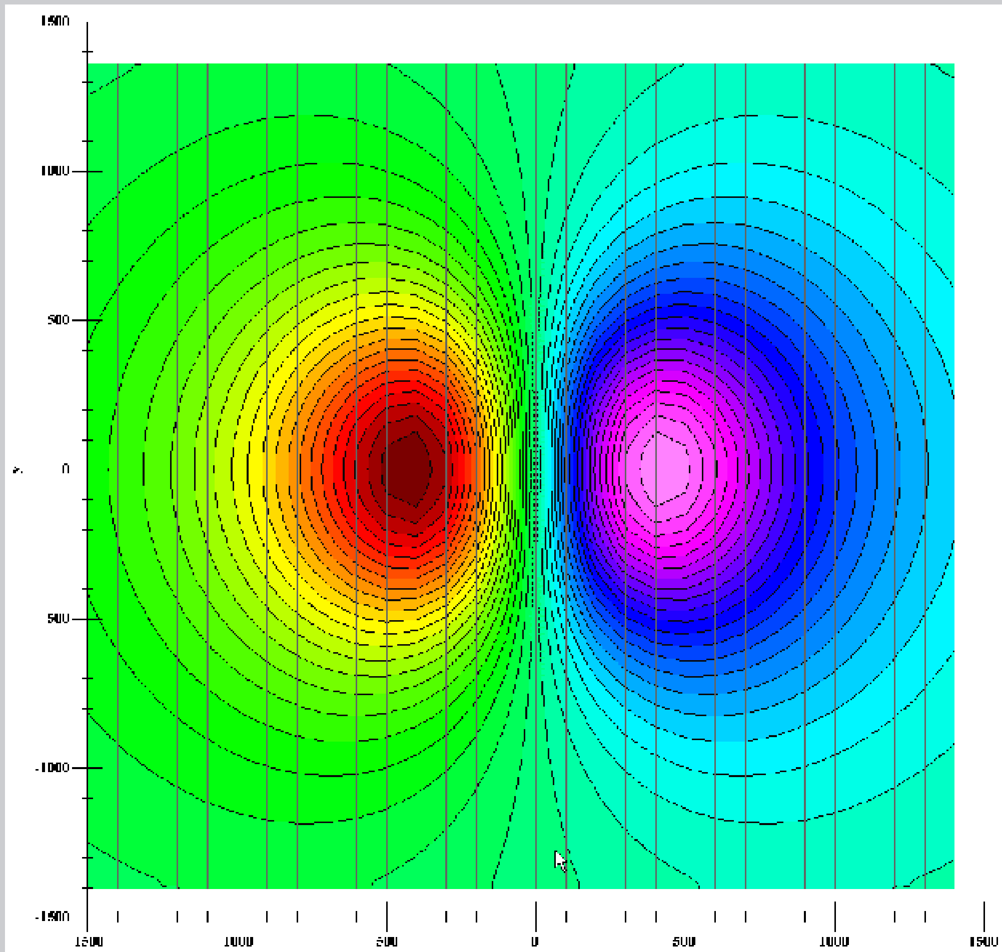


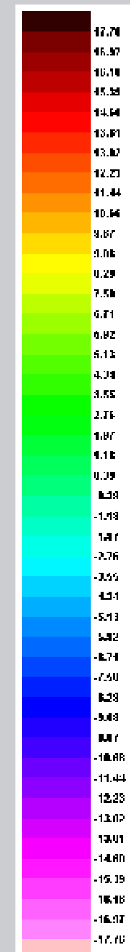
Seg2005_1.dat

Computed Model - gradients of vertical gravity Gz

Synthetic across-line gradient

“True_dat”
a_dome

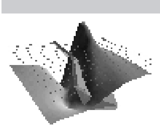




E.O.

Seg2005_115.dat

1 E.O. (Eotvos**) = 1e-4 mGal/m**

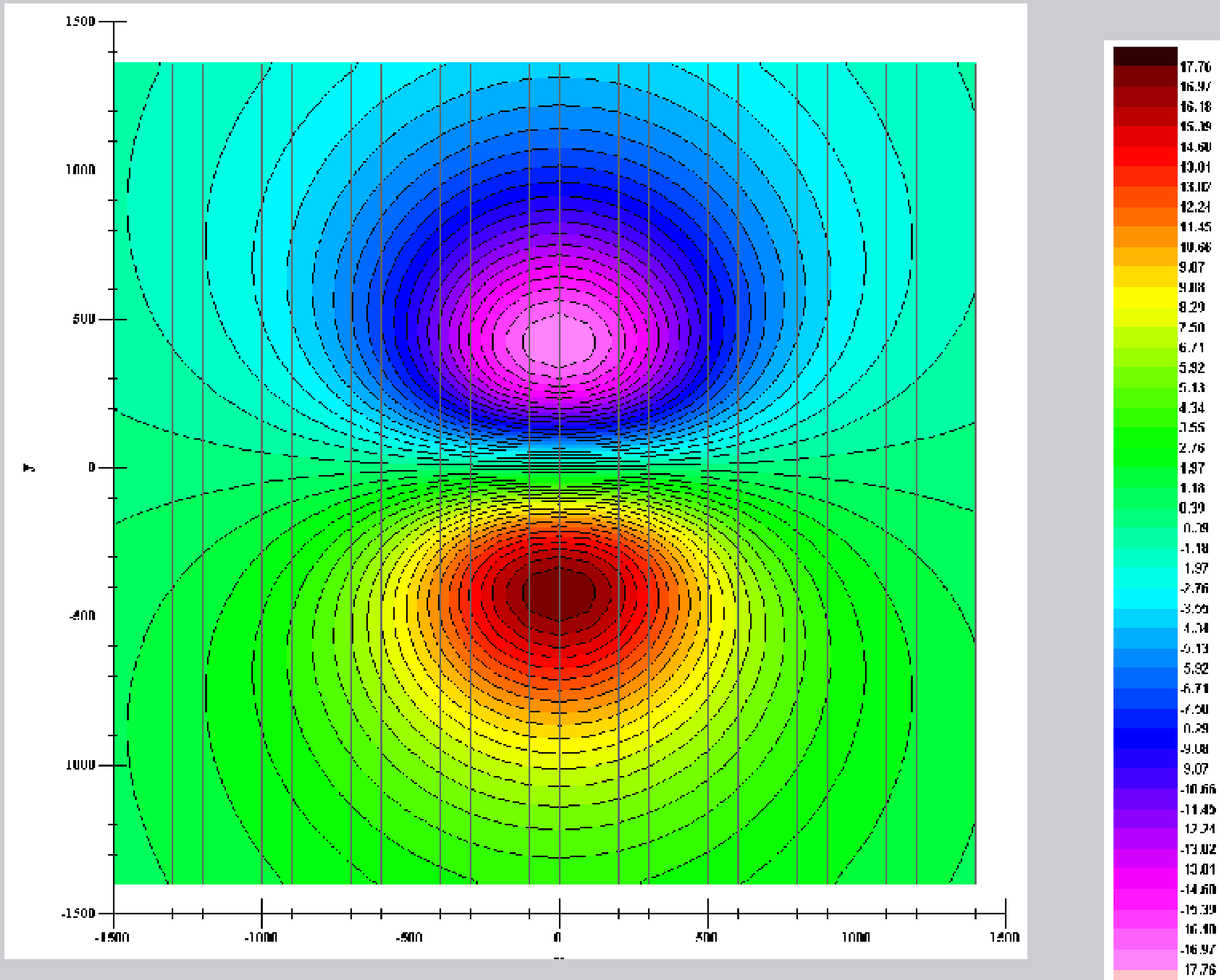


BHL

Computed Model - gradients of vertical gravity Gz

True_data
_dome

Seg2005_1
15.dat



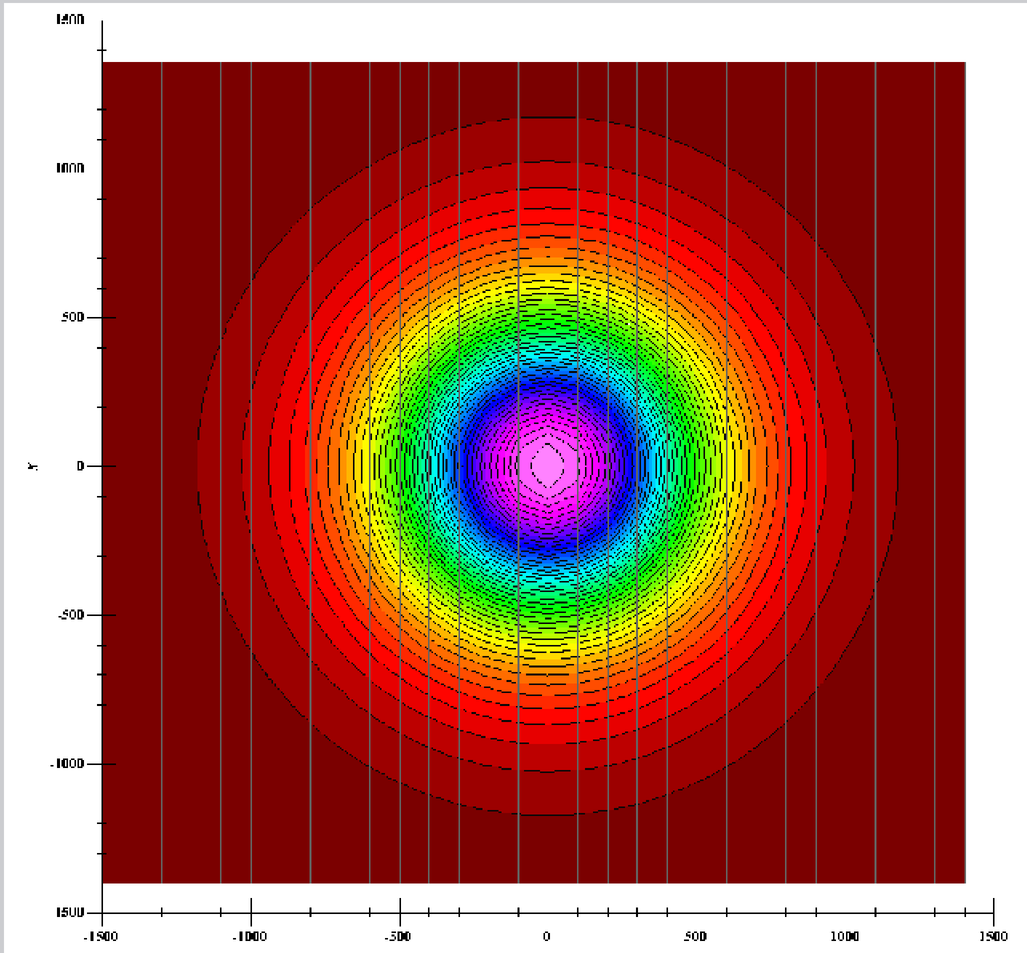
Synthetic inline gradient

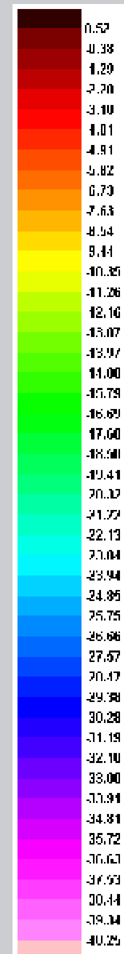
E.O.

Computed Model - gradients of vertical gravity Gz

True_data
_dome

Seg2005_1
15.dat





Synthetic vertical gradient

E.O.

Euler Deconvolution- setting and data types

- **Euler Deconvolution Setting:**
  - ❖ **Moving window size: 750 m by 750 m**
  - ❖ **Structural index range: 0.25 - 2.25**

  (excluding 0 as there is no true anomaly with this rate)

0 - sill/dyke/step

0.5 - ribbon

1 - pipe

2 - sphere

- **3 types of data:**
  - ➢ ***Quasi-analytic Gz with quasi-analytic gradients***

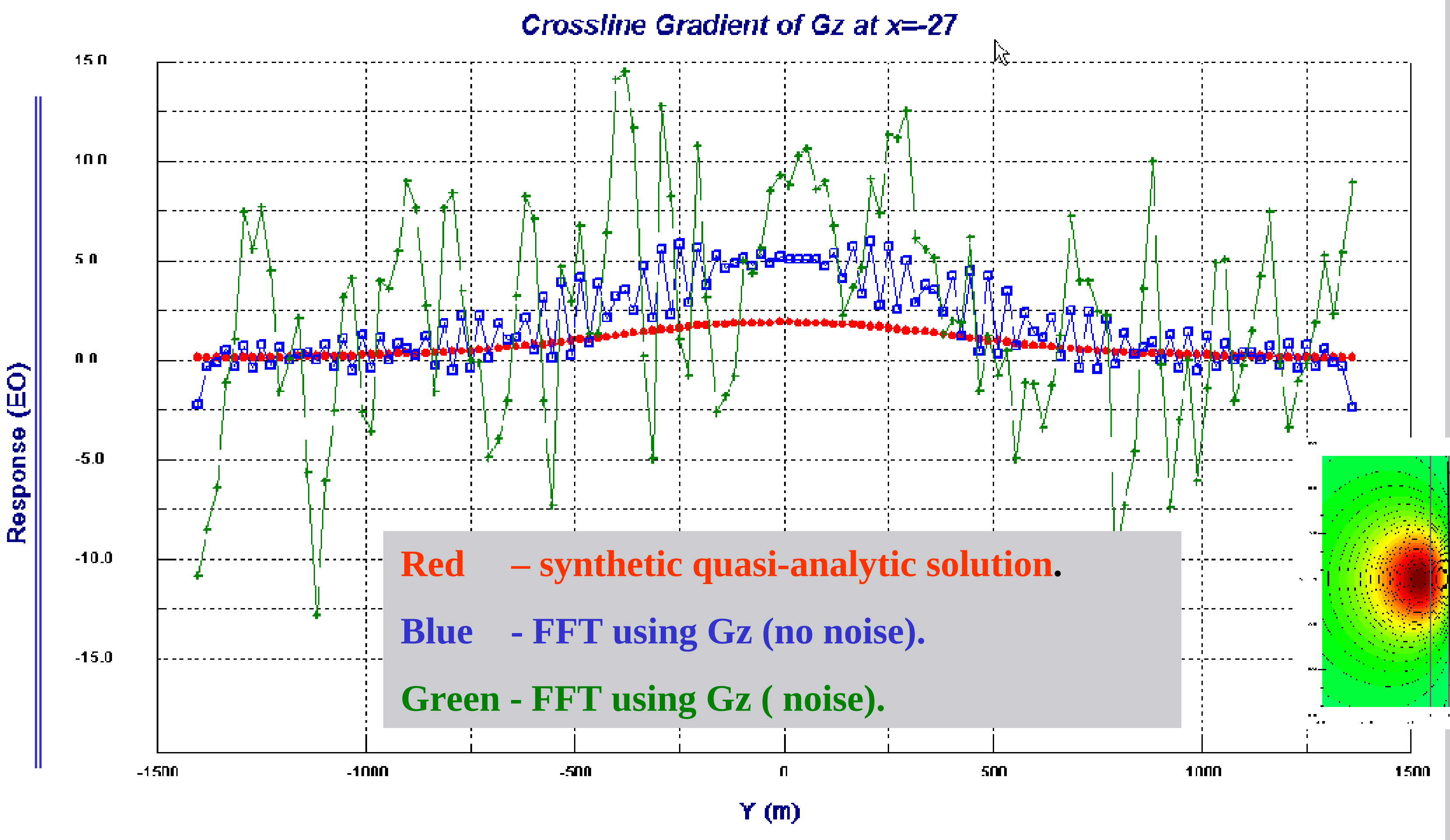
    interpolated on grid with cell size 20m by 50m (1/2 data)
  - ➢ ***Quasi-analytic Gz and FFT gradients***

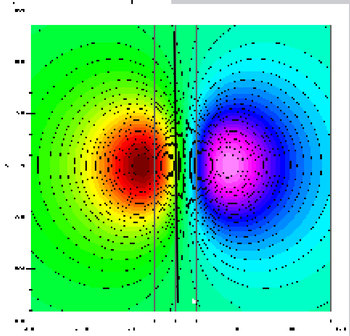
    FFT grid cell size 21m by 46 m
  - ➢ ***Quasi-analytic Gz with noise and FFT gradients***

    FFT grid cell size 21m by 46 m

    Gaussian noise with S.D. = 0.05mgal (.1% of peak)

Euler Deconvolution - Synthetic VS. FFT Gradients

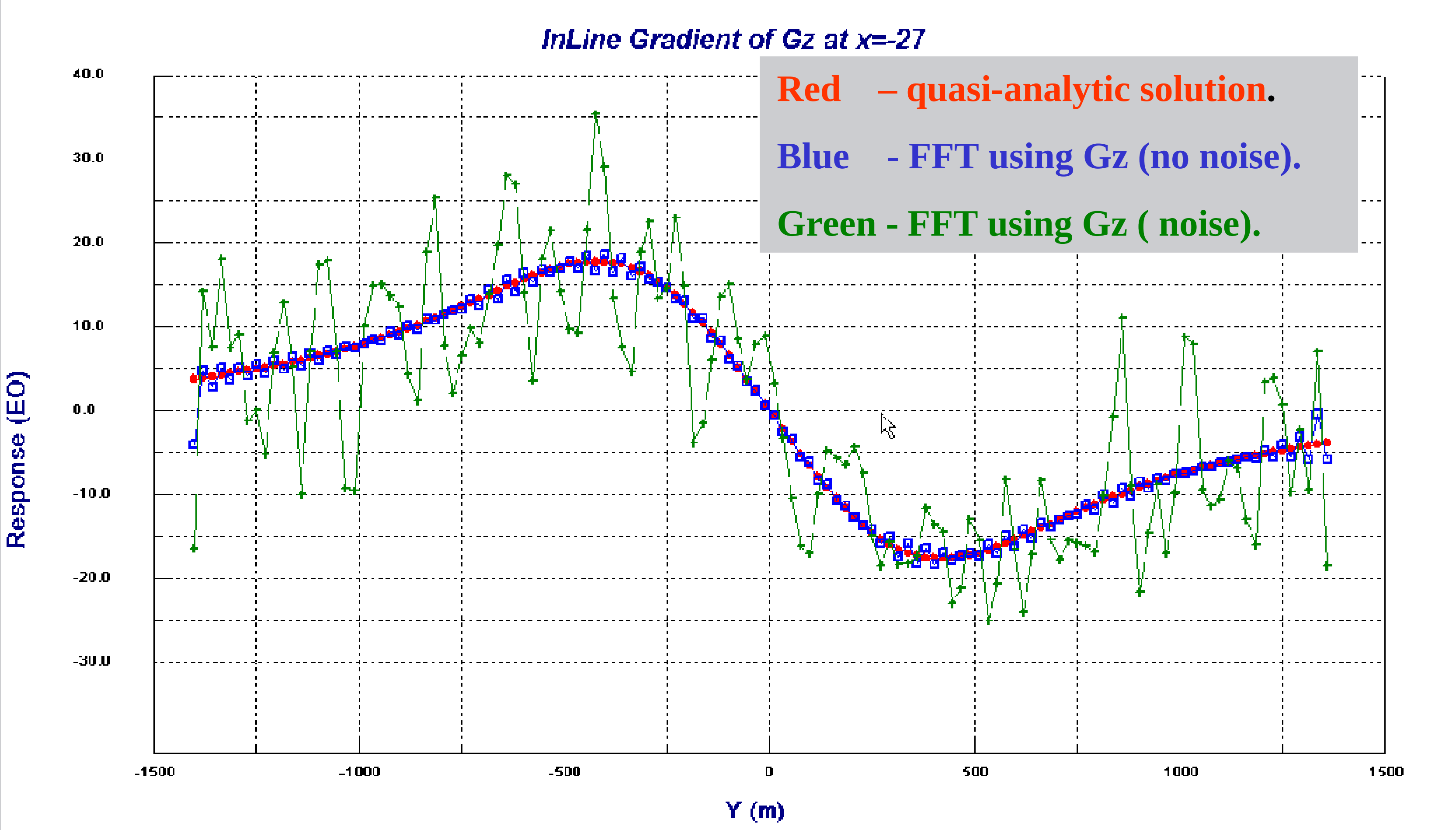




Across-line gradient along a NS profile at east –27m.

Seg2005_125.dat, Seg2005_124.dat, Seg2005_126.dat

Euler Deconvolution - Synthetic VS. FFT Gradients

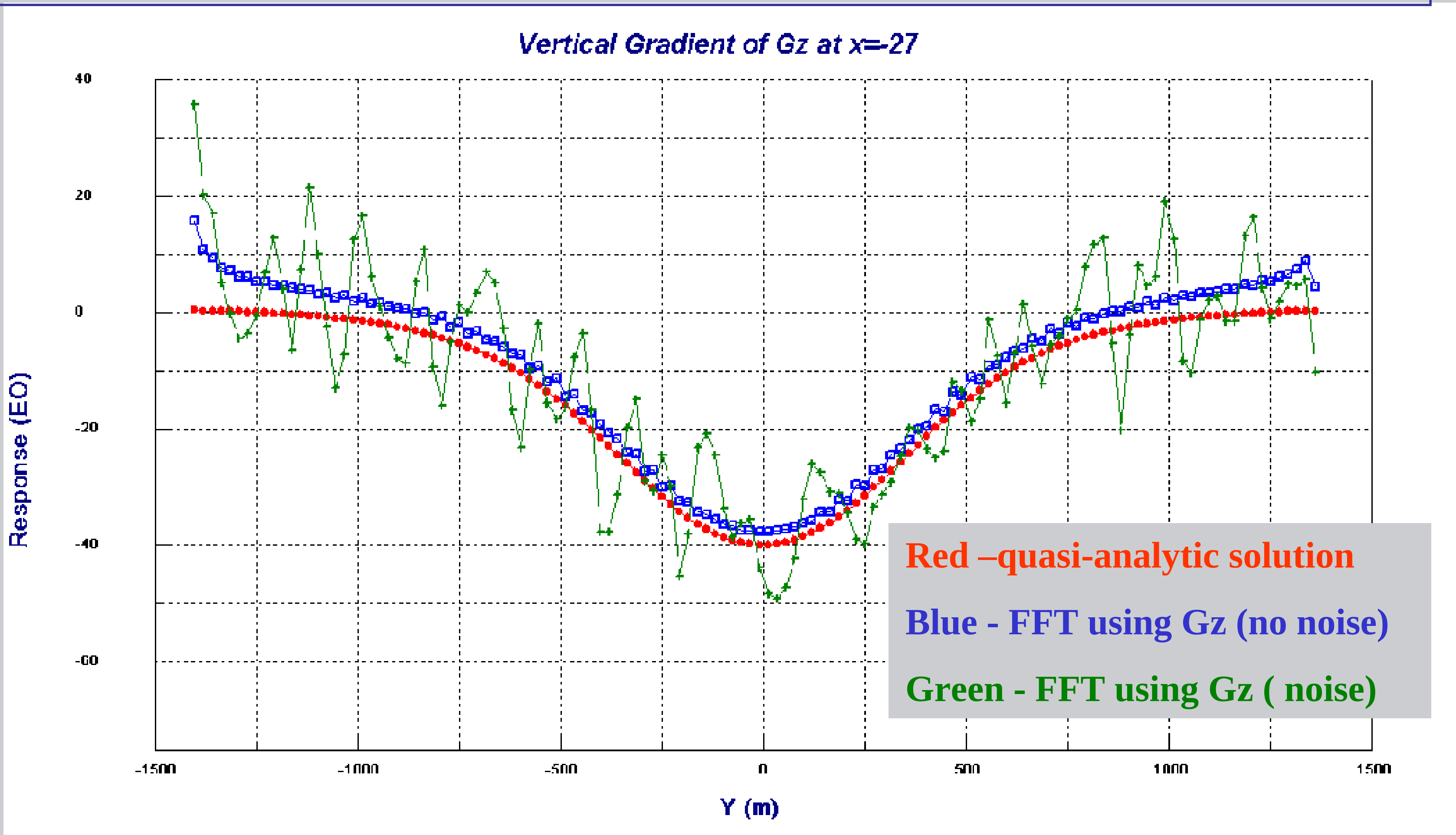


tighter inline grid due to higher inline sampling

Seg2005_125.dat, Seg2005_124.dat, Seg2005_126.dat

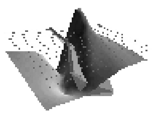
BHL

Euler Deconvolution - Synthetic VS. FFT Gradients



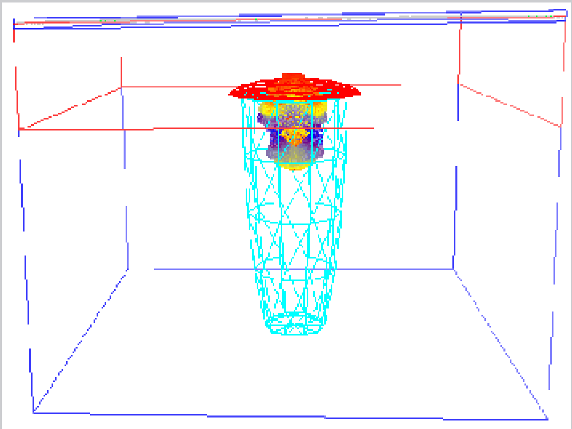
Seg2005_125.dat, Seg2005_124.dat, Seg2005_126.dat

incorporates errors of crossline

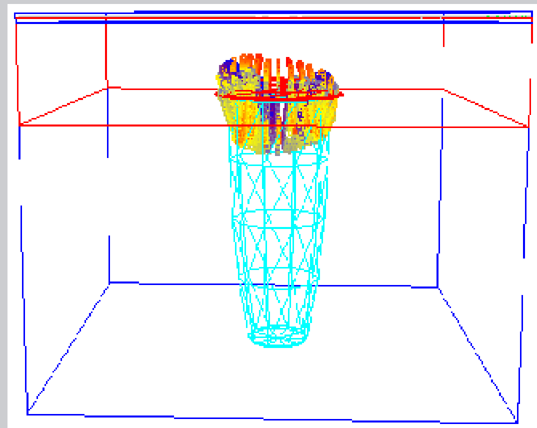


Euler Solutions-comparison

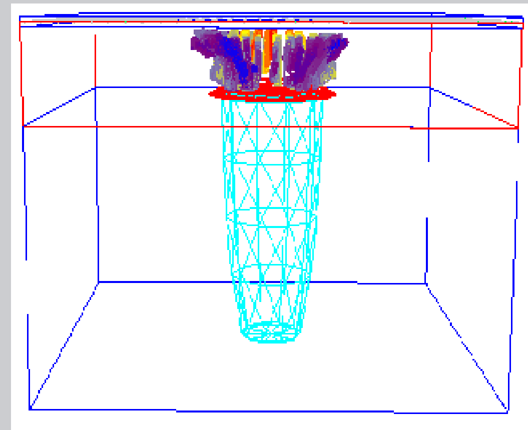
Note: after post-processing



Quasi-Analytic Gz
with Quasi-Analytic gradients



Quasi-Analytic Gz
with FFT gradients



Quasi-Analytic Gz
with noise and FFT gradients

Colors indicate the deviation of solution

Red: low, Yellow: medium, Blue: high

Seg2005_90.dat, Seg2005_88.dat, Seg2005_93.dat

How to generate gradients effectively?

As seen, it is essential to have accurate gradients in order to generate good Euler solutions.

The problem is that FFT gradients are not reliable in the case that the line spacing is “large”.

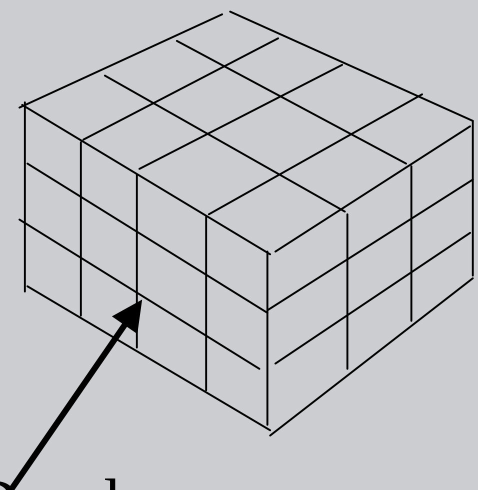
Next we will demonstrate that accurate gradients can be generated utilizing 3D-inversion (reference???).

3D-Gravity Inversion-theory

Inversion Model

Minimization Technique:

Conjugate Gradient



3D-volume

size ?, depth ?

Minimize $\left|\phi - \phi^*\right|$

Subject to $l_i \le \rho_i \le u_i$, $i = 1, 2, \cdots, N$

$$\phi = \left\| W_d (d - \hat{d}) \right\|^2$$

$$d = d(\rho_1, \rho_2, \cdots, \rho_N)$$

ϕ^* prescribed tolerance

$(\hat{d}_1, \hat{d}_2, \cdots \hat{d}_M)$ measured data

$(d_1, d_2, \cdots d_M)$ simulated data

$$W_d = diag(1/\sigma_1, 1/\sigma_2, \cdots, 1/\sigma_M)$$

σ_i is the standard deviation of the i-th datum

3D-Gravity Inversion-synthetic data

- **Apply inversion to Gz of salt dome**
- **Add random noise to Gz**

  - σ = 0.05 mGal

- **Initial 3D-grid volume**

  - dimensions = 3200 m x 3000 m x 2000 m (lx,ly,lz)

- **The distance from top of the 3D-volume to measurement points**

  - 50 m.

- **Grid cell**

  width 100m in easting , 50m in northing, and vertical extent 100m

- **Utilize the data deviation during the inversion - $\chi 2$**

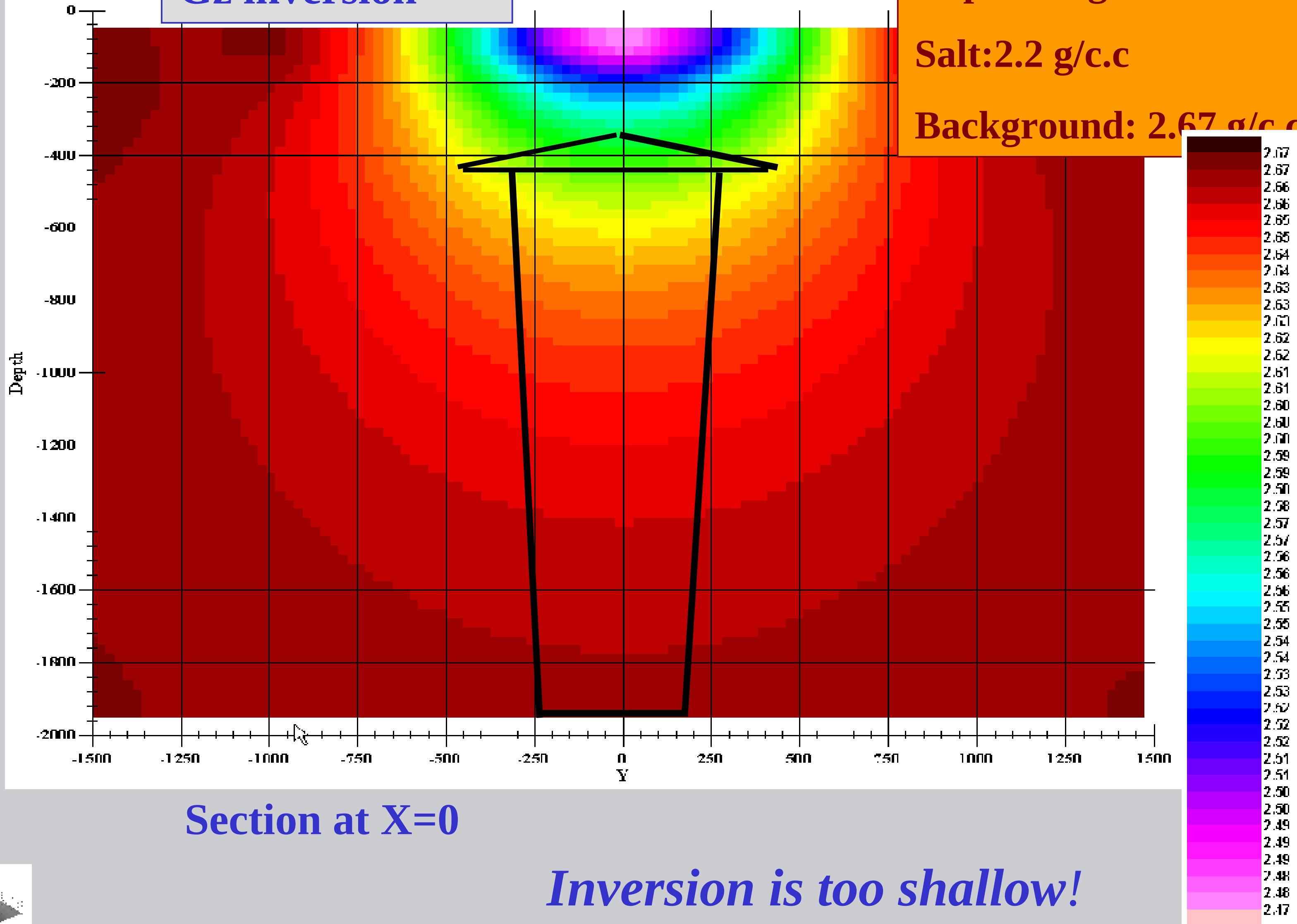
3D-Gravity Inversion-inverted model

Gz inversion

Cap: 2.55 g/c.c.

Salt:2.2 g/c.c

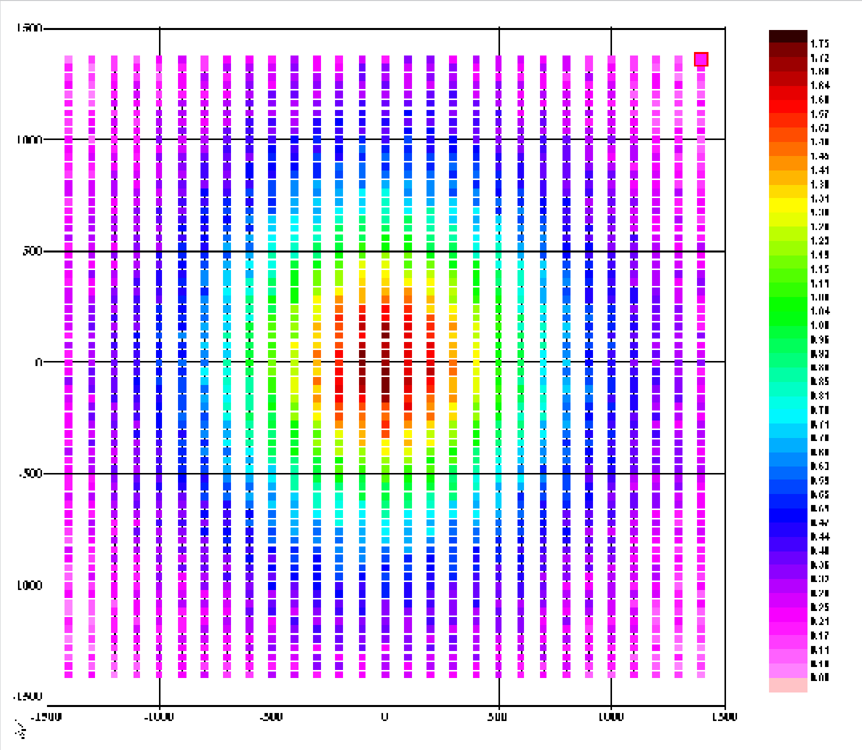
Background: 2.67 g/c.c



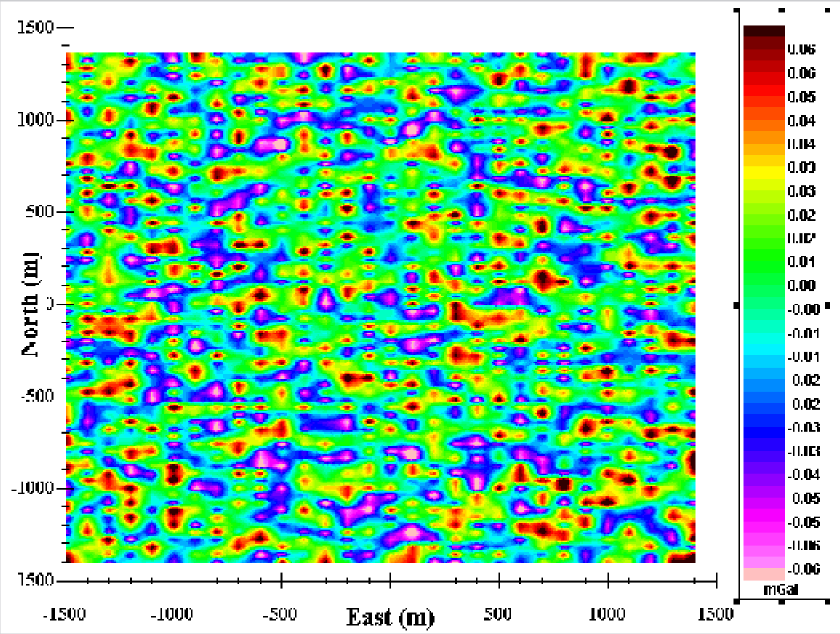
Section at X=0

Inversion is too shallow!

3D-Gravity Inversion-misfit



Computer Gz with Noise



Gz inversion misfit ie. relatively random.

Seg2005_102.dat

Seg2005_115.dat

Euler Deconvolution-utilization of inversion results

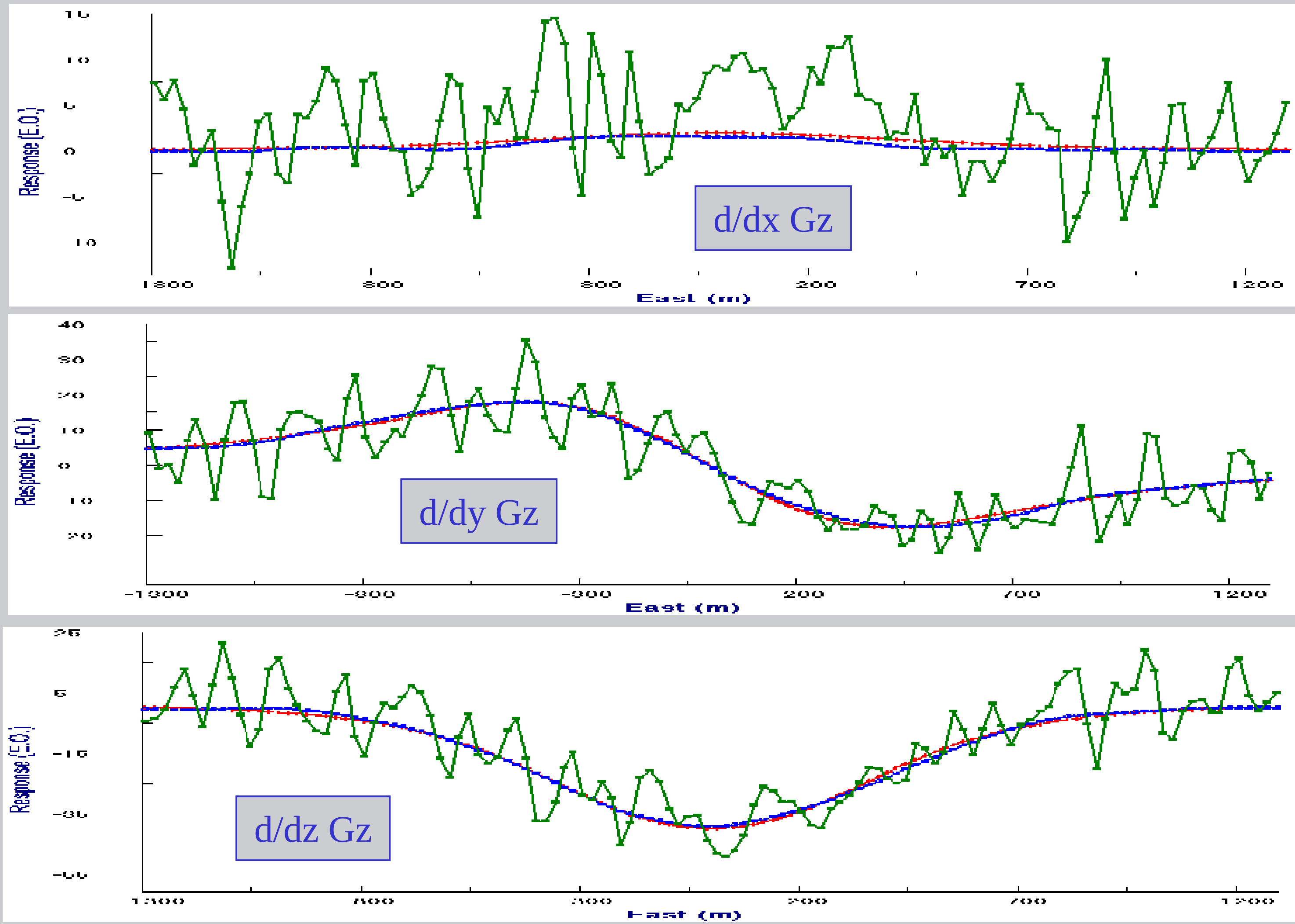
EQUIVALENT SOURCE TECHNIQUE

Generate Gz and its gradients from inverted model

- **Apply Euler Deconvolution to these data**
  - **Moving window size: 750 m by 750 m**
  - **Structural index range: 0.25 - 2.25**

Objective: Generate more accurate Euler solutions as starting model for inversion.

Comparison of Gradients- along NS profile at easting –27m.



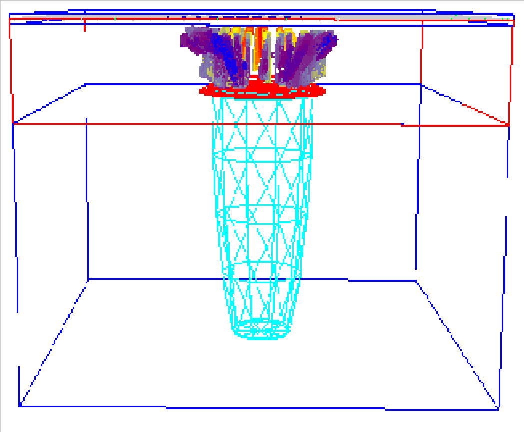
Red – Computed Solution.

Seg2005_125.dat, Seg2005_127.dat, Seg2005_126.dat

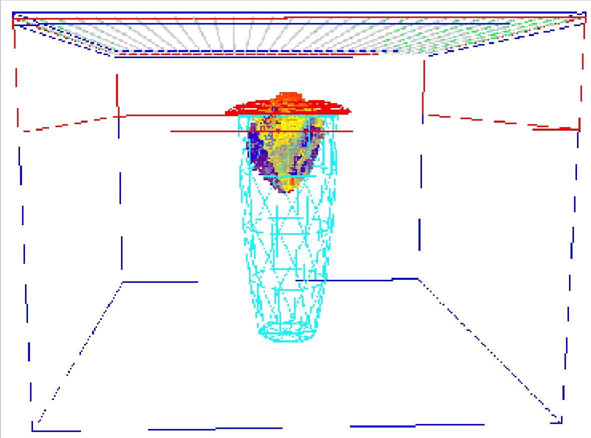
Blue - Equivalence Source - i.e. computed from inversion model using Gz (noise)

Green – FFT using Gz (noise).

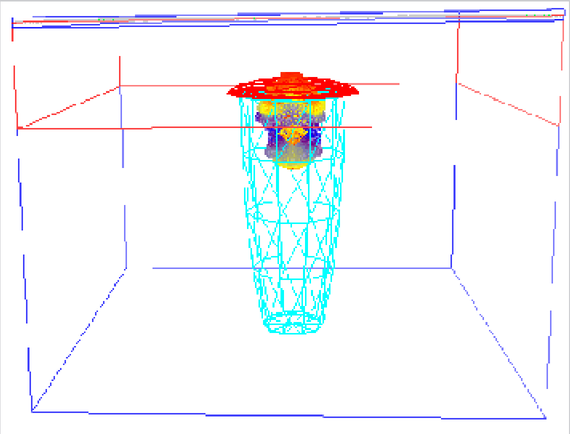
Euler Solutions-comparison



from FFT using data with noise



by Equivalent source using data with noise



from computed gradients by analytic solution

Conclusion:

Euler solutions can be improved

with the use of inverted gradients

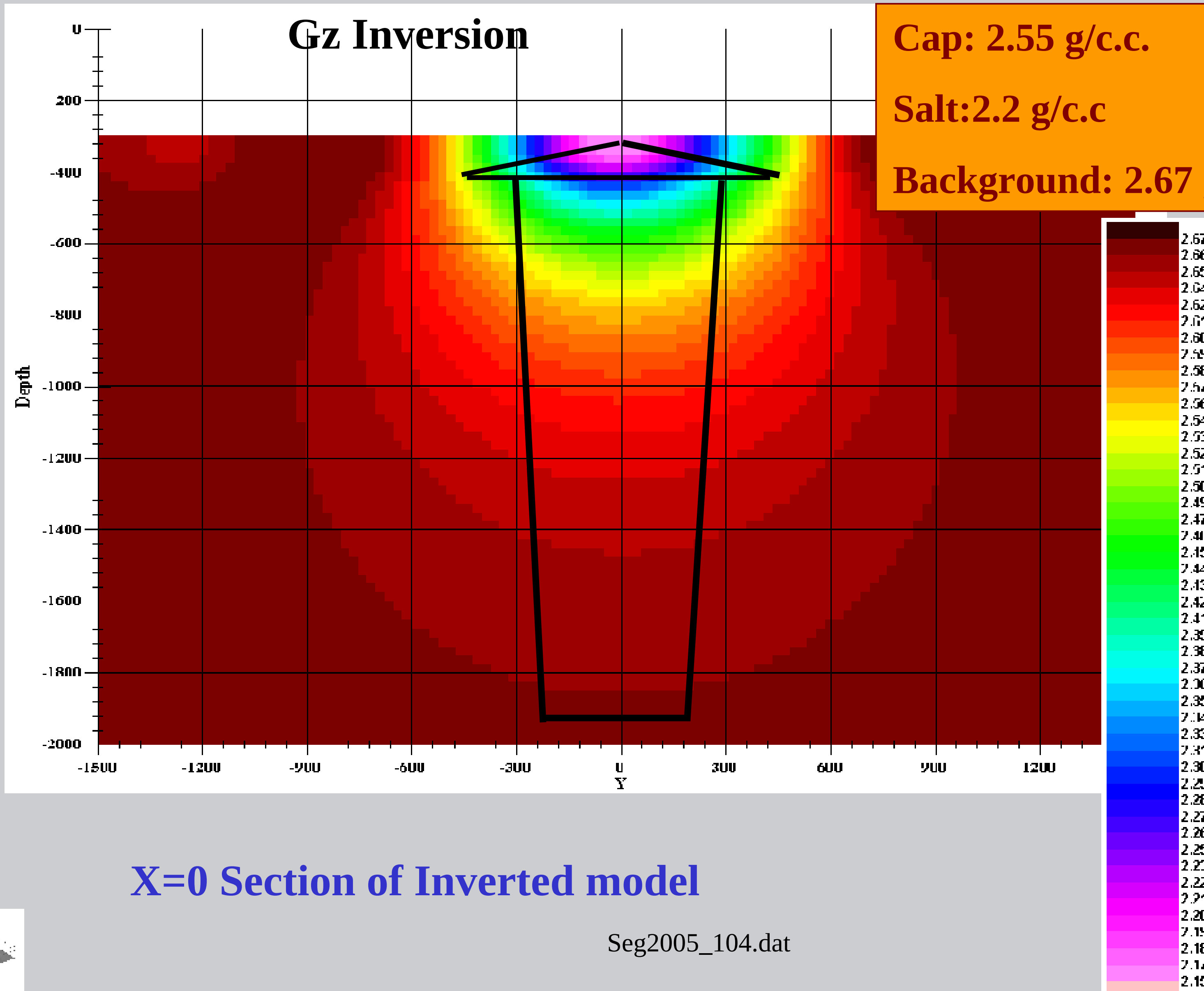
Seg2005_93.dat, Seg2005_111.dat, Seg2005_90.dat

3D-Gravity Inversion- Utilization of Euler solution information

Cap: 2.55 g/c.c.

Salt:2.2 g/c.c

Background: 2.67 g/c.c

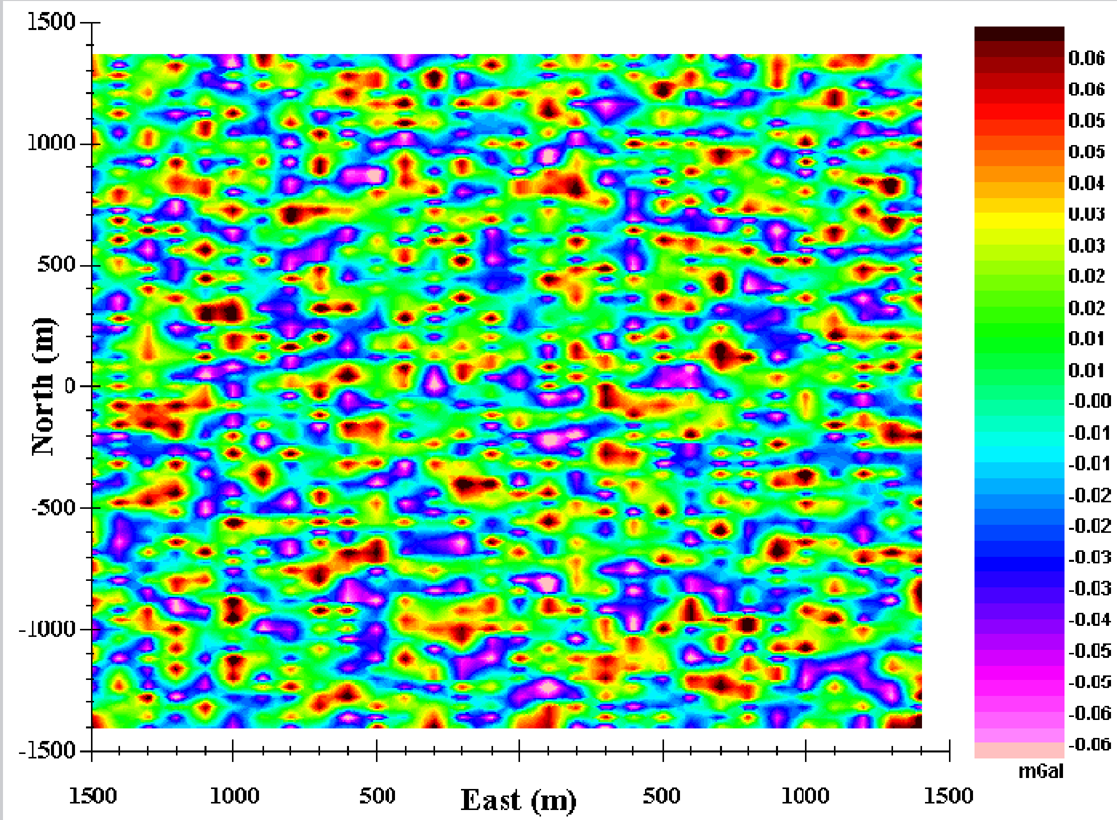


X=0 Section of Inverted model

Seg2005_104.dat

BHL

3D-Gravity Data Inversion-misfit



Gz inversion misfit.

The top of 3D- grid volume is 300 m from ground surface.

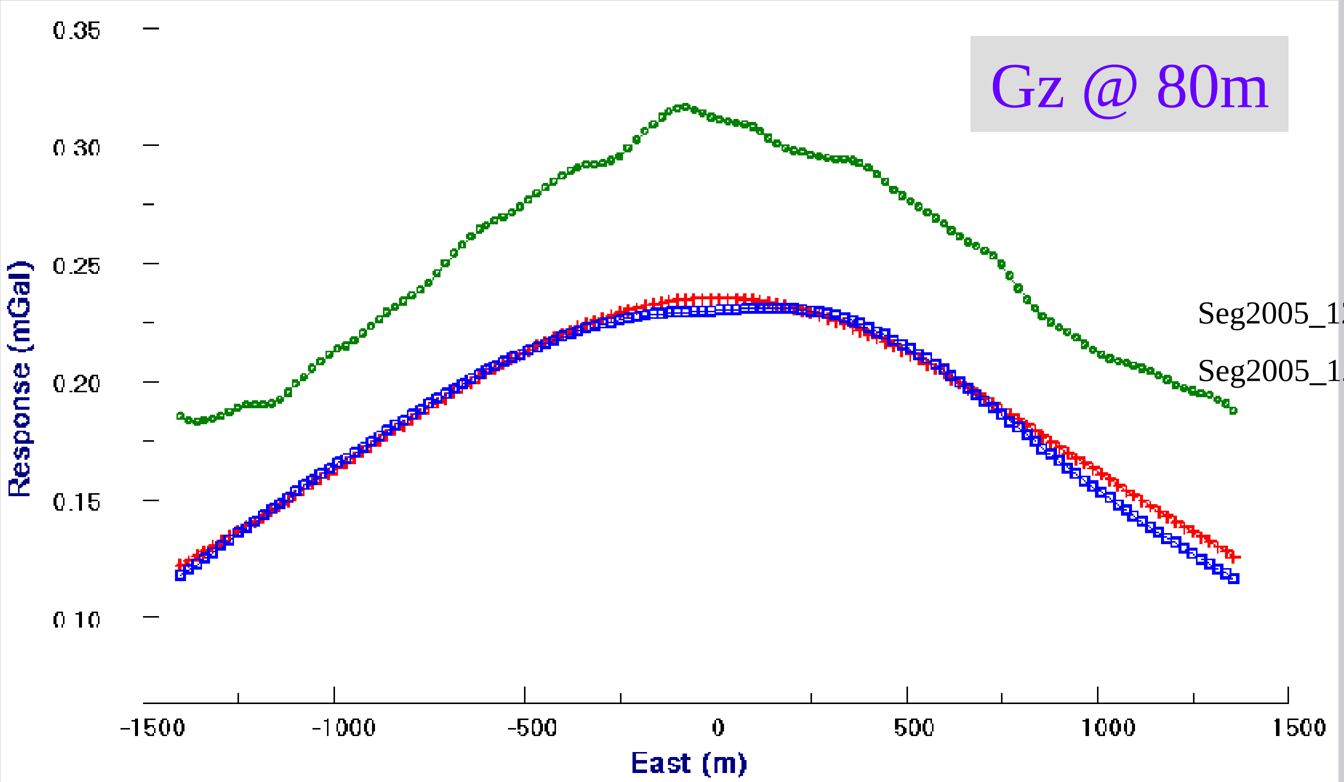
Seg2005_107.dat Seg2005_115.dat

Upward continuation –utilization of 3D-Inversion

Purpose: to demonstrate that equivalent source technique can be used for upward continuation

1. Compute the responses of the dome at 80m above the ground using
2. perform upward continuation by standard FFT
3. perform upward continuation by Equivalent source technique
4. Compare with analytic solutions

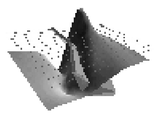
Upperward continuation –utilization of 3D-Inversion



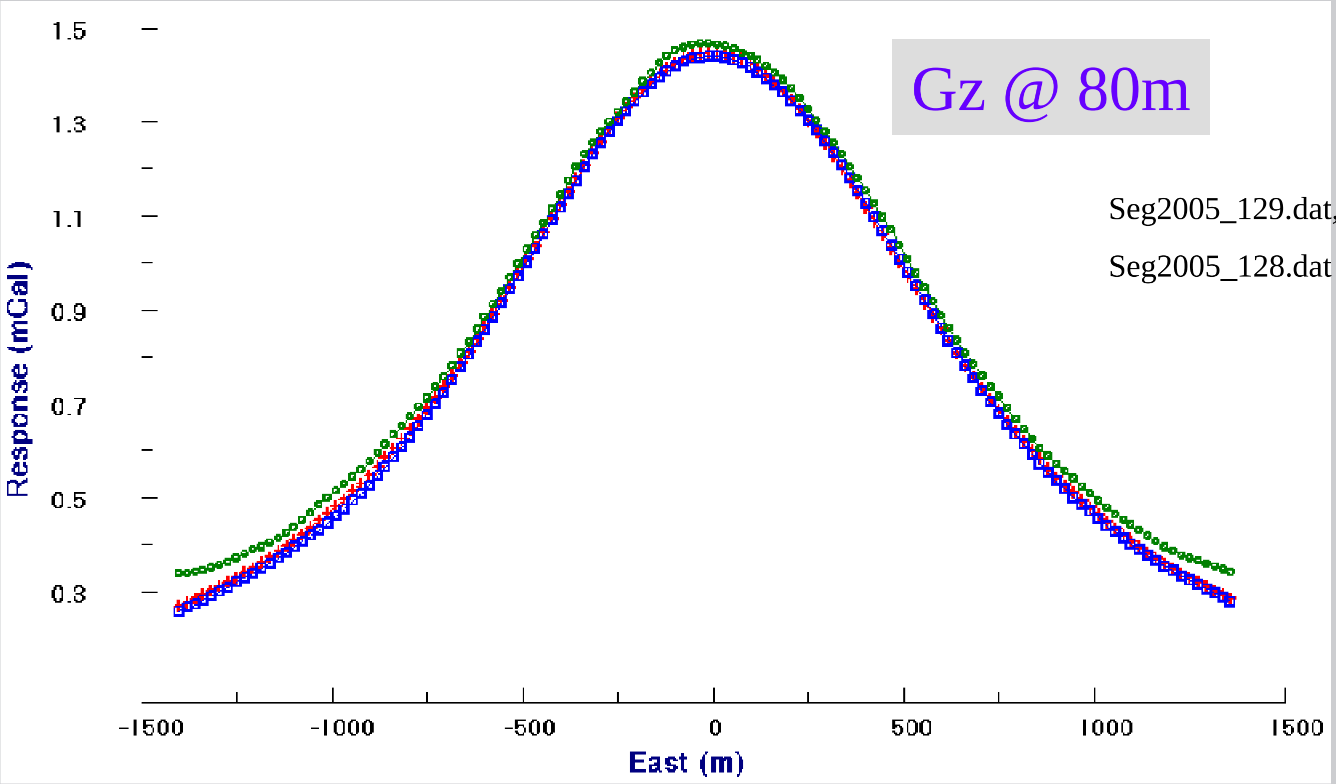
Seg2005_129.dat, Seg2005_130.dat, Seg2005_128.dat

Red: **“true” solution.** **Blue: (via. Equivalence Source).** **Green: via FFT**

Along NS profile at x=1500m Flight height: 80m



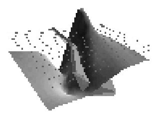
Upperward continuation –utilization of 3D-Inversion



Seg2005_129.dat, Seg2005_130.dat, Seg2005_128.dat

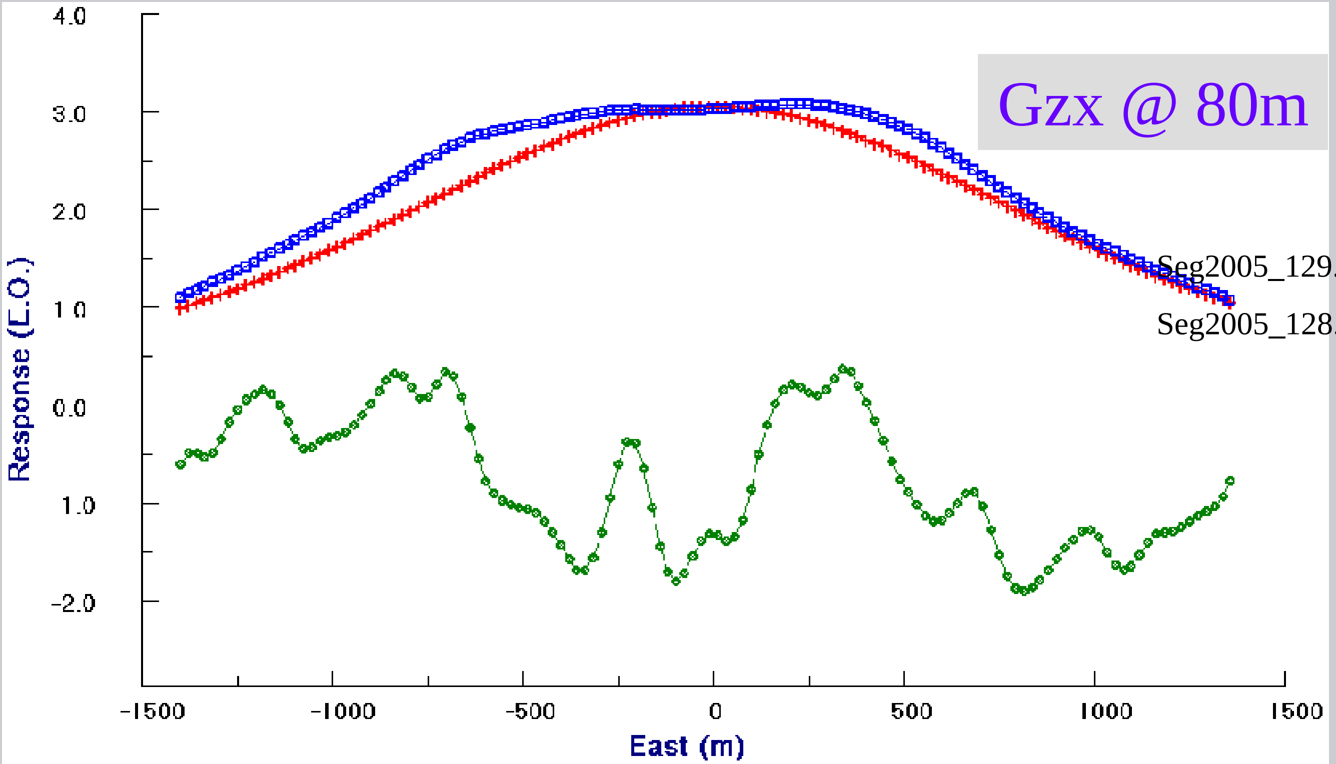
Red: **"true" solution.** **Blue: (via. Equivalence Source).** **Green: via FFT**

Along NS profile at x=27m. ***Flight height: 80m***



BHL

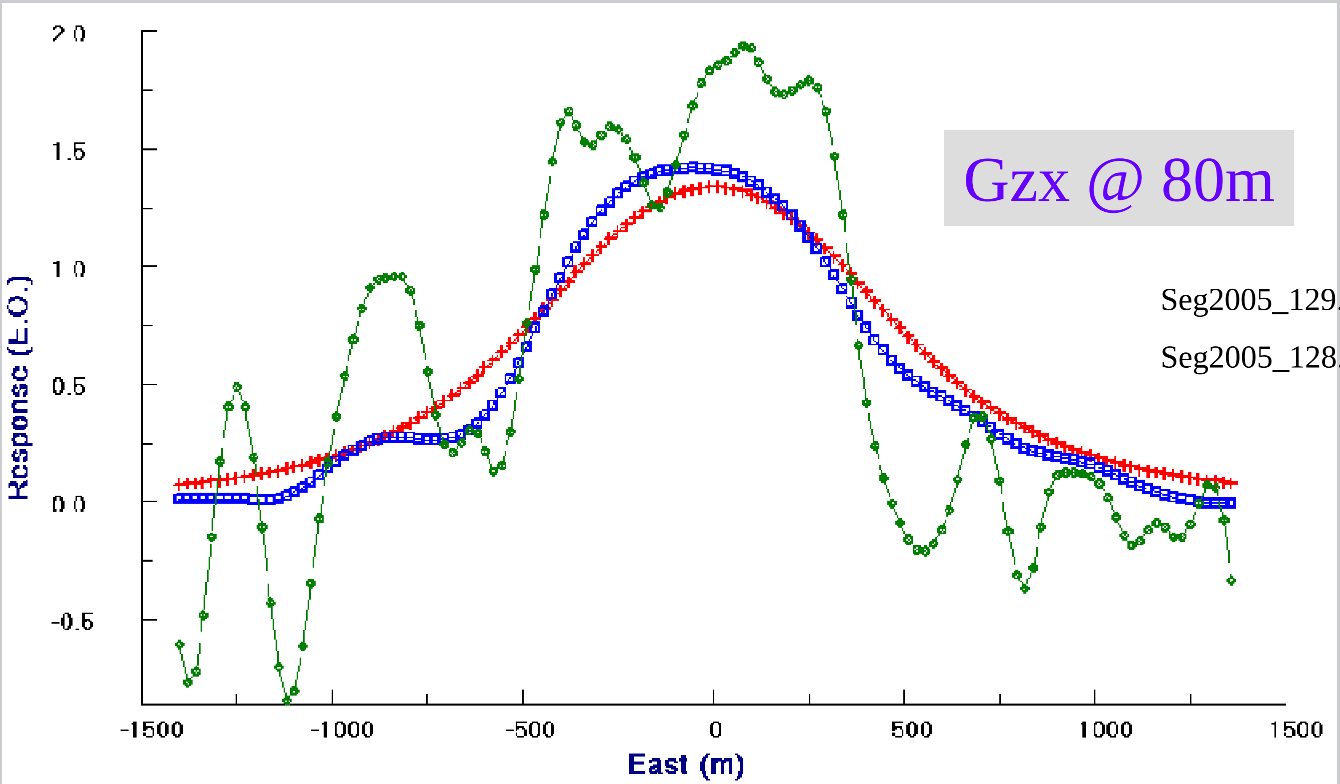
Upward continuation –utilization of 3D-Inversion



Red: **“true” solution.** **Blue: (via. Equivalence Source).** **Green: via FFT**

Along NS profile at x=1500m ***Flight height: 80m***

Upperward continuation **–utilization of 3D-Inversion**



Seg2005_129.dat, Seg2005_130.dat,

Seg2005_128.dat

Red: **“true” solution.** **Blue: (via. Equivalence Source).** **Green: via FFT**

Along NS profile at x=27m. ***Flight height: 80m***

Conclusions

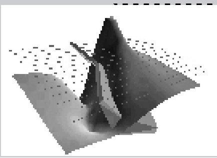
Based on inverted models, various components of gravity/gradient data can be generated accurately by the equivalent source technique.

❖ Gravity/gradient data generated with inverted models can greatly enhance the Euler Deconvolution technique.

❖ Euler solutions can provide useful information for setting starting models of the inversion.

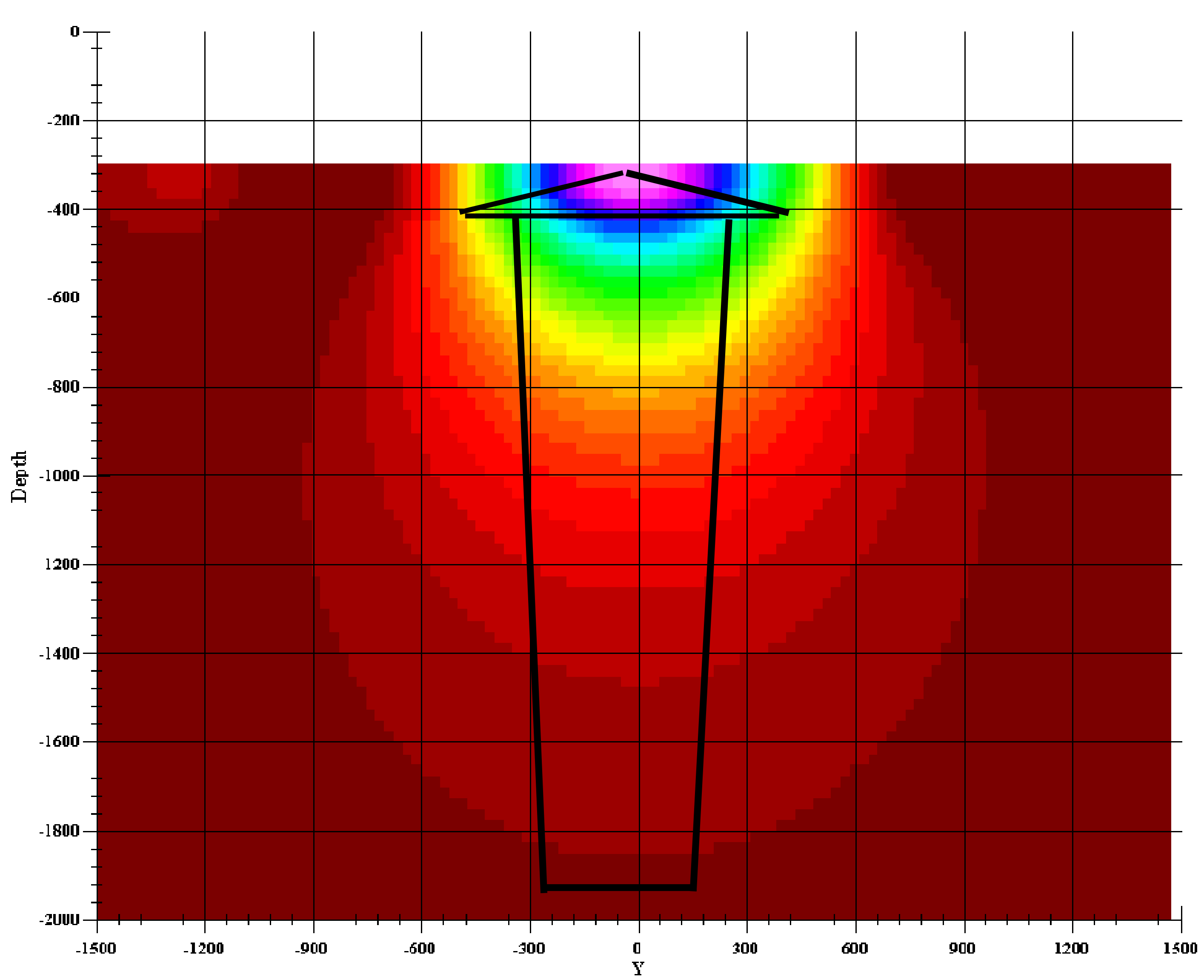
❖ Utilization of data deviation gives better inversion results

❖ The equivalent source technique can provide better results than standard FFT for derivatives and continuation (filtering?)



Direction:

❖Test equivalent source technique with real airborne gradient data.



0
-200
-400
-600
-800
-1000
-1200
-1400
-1600
-1800
-2000
Depth
-1500
-1200
-900
-600
-300
0
300
600
900
1200
1500
Y