

3 Examples of *PetRos EiKon* research as pertains to the use of measured TMI derivatives

Example 1: How should gradients be processed?

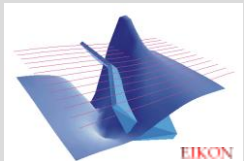
Example 2: When should gradients be collected?

Example 3: Will measured gradients increase my data resolution?

• Example 1

3 Sensor Mag Processing: Derivation of TMI Gradients

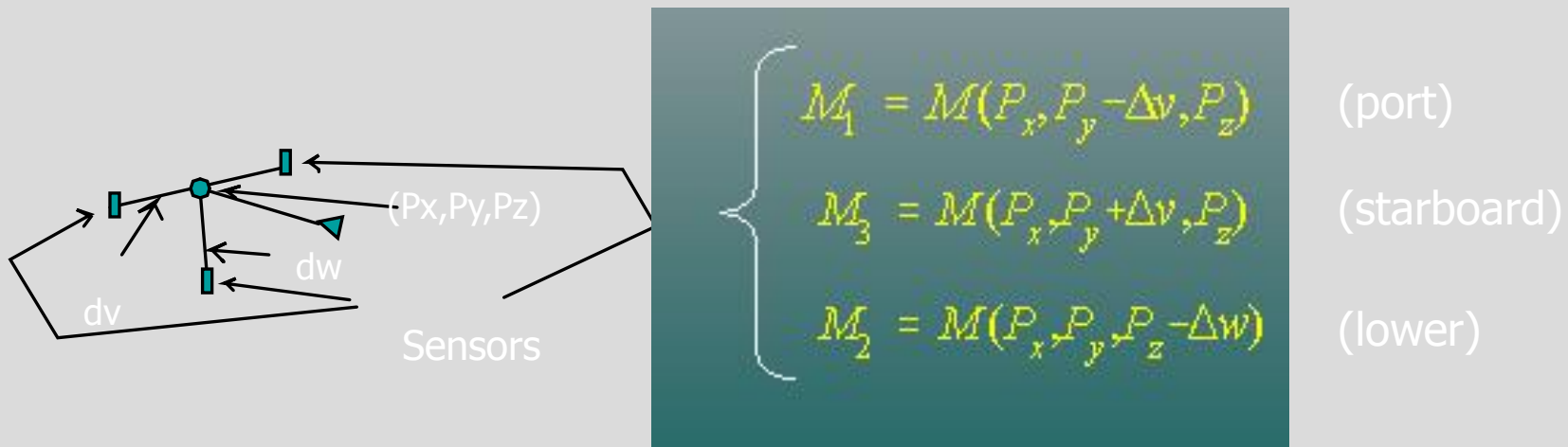
*Algorithms for de-rotating
2 horizontally offset magnetometers
with 1 vertically offset magnetometer*



In this configuration, the 3 sensors are set on a rigid frame which varies its orientation continuously during flight. The problem is how to obtain gradients in some useful and consistent coordinate system.

Locally (ie. at each position), the sensors measure derivatives in somewhat random orientations. Unless, the gradients can be de-rotated to a consistent frame then they have limited usefulness.

3 TMI measurements at each station in the local frame originating at (P_x, P_y, P_z)



Grid system vs. Local

Normally gradient vectors can be orientated easily from a local frame to a more general geographic, geomagnetic or grid system if the orientation of the rigid system and the 3 gradient vectors are known. Mathematically,

$$\frac{\partial M}{\partial x} = \frac{\partial M}{\partial u} \frac{\partial u}{\partial x} + \frac{\partial M}{\partial v} \frac{\partial v}{\partial x} + \frac{\partial M}{\partial w} \frac{\partial w}{\partial x}$$

(u,v,w) local

(x,y,z) Grid or Geographic

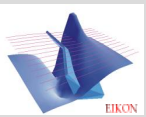
$$\frac{\partial M}{\partial y} = \frac{\partial M}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial M}{\partial v} \frac{\partial v}{\partial y} + \frac{\partial M}{\partial w} \frac{\partial w}{\partial y}$$

$$\frac{\partial M}{\partial z} = \frac{\partial M}{\partial u} \frac{\partial u}{\partial z} + \frac{\partial M}{\partial v} \frac{\partial v}{\partial z} + \frac{\partial M}{\partial w} \frac{\partial w}{\partial z}$$

The orientation is represented mathematically by:

$$\frac{\partial(u,v,w)}{\partial(x,y,z)}$$

i.e. pitch, roll, heading



Two of the local derivatives can be derived via the following equations:

$$\frac{\partial M}{\partial v} = \frac{M_3 - M_1}{2\Delta v}$$
$$\frac{\partial M}{\partial w} = \frac{\frac{1}{2}(M_1 + M_3) - M_2}{\Delta w}$$

Local Transverse Gradient

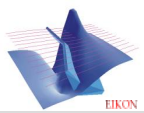
Local Vertical Gradient

At each data point, we wish to recover the 3 gradients in the grid system. i.e., $\delta M / \delta x$, $\delta M / \delta y$, and $\delta M / \delta z$. However,

→ three equations in four unknowns

$$\frac{\partial M}{\partial u}, \frac{\partial M}{\partial x}, \frac{\partial M}{\partial y}, \frac{\partial M}{\partial z}$$

!!??



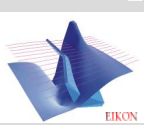
In-Line Derivative

The derivative in the direction of flight (in-line) has no direct measurement for this configuration. Normally, it is estimated by the difference in 2 in-line average measurements, i.e.

$$\frac{\partial M}{\partial u} \approx \frac{\overline{M}_1(x_1, y_1, z_1) - \overline{M}_2(x_2, y_2, z_2)}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 - (z_1 - z_2)^2}}$$

Where, $\overline{M}_1, \overline{M}_2$ are the average M at each data point.

However, using this estimate of the in-line derivative with the other local derivatives to de-rotate the gradients as if in a fixed rigid system presents problems. As will be seen later.



Directional Derivative

Another approach is to use the notion of a directional derivative

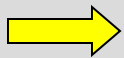
Flight Path

$$A = (P_x, P_y, P_z) - (P'_x, P'_y, P'_z)$$

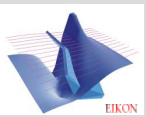
Previous observation at (P'_x, P'_y, P'_z)

$$A_x \frac{\partial M}{\partial x} + A_y \frac{\partial M}{\partial y} + A_z \frac{\partial M}{\partial z} = \bar{M} - \bar{M}'$$

$$\left\{ \begin{array}{l} \bar{M} = \frac{1}{3}(M_1 + M_2 + M_3) \\ \bar{M}' = \frac{1}{3}(M'_1 + M'_2 + M'_3) \end{array} \right.$$



fourth equation !

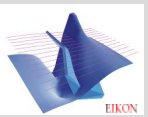


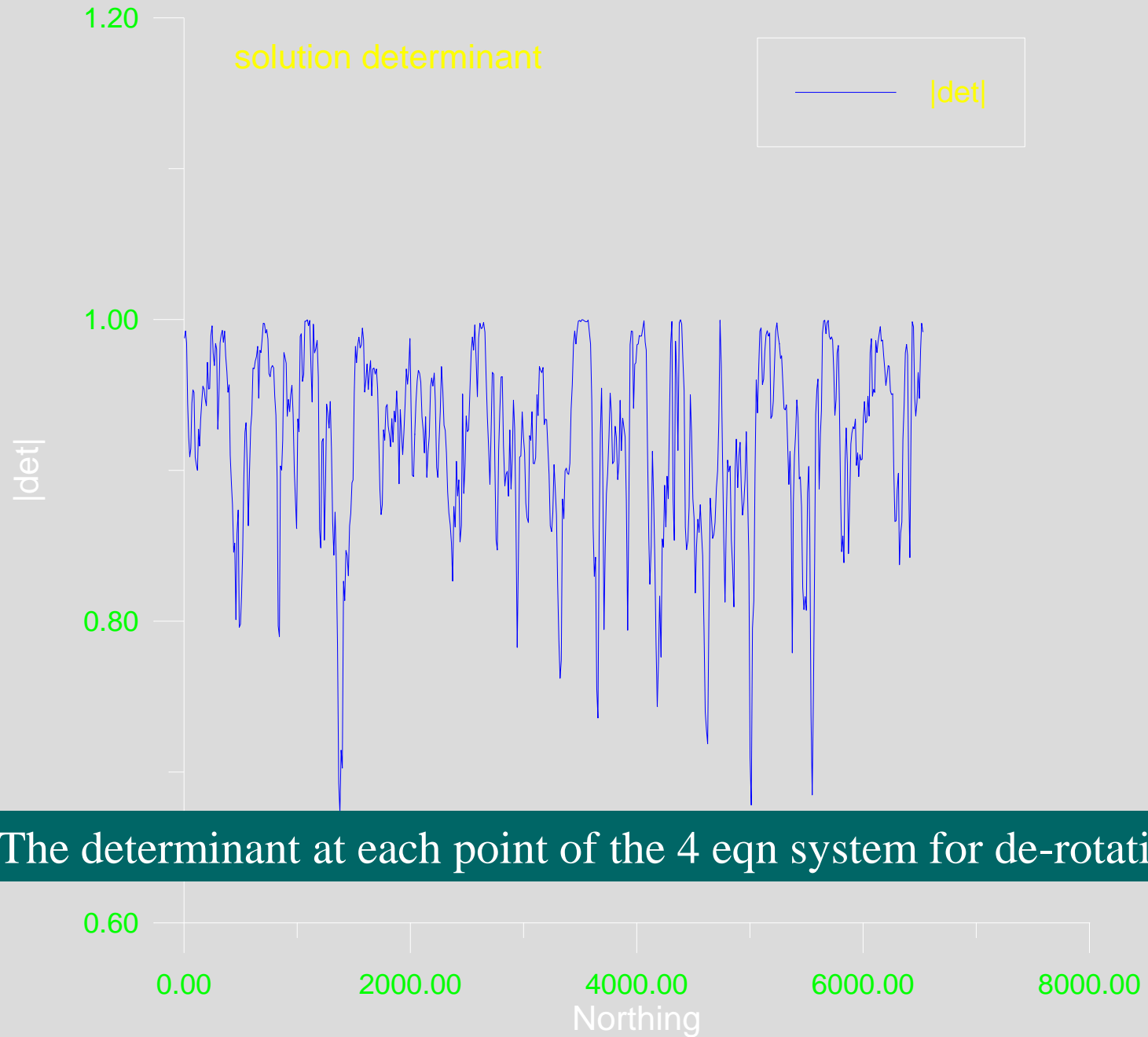
Synthetic (nonphysical) Data Example

Simulated Flight Path

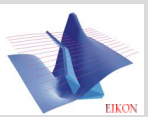
- *roll, pitch, heading, altitude variations*
- *Magnetometer orientations generated semi-randomly*
- *Simulated data mathematically generated allowing analytic gradients for benchmarking*

•In this example, the locations of the center of the sensors vary pseudo-randomly along a prescribed flight path and the orientations of the 3-sensor rigid system are allowed to vary at each data position in a smooth but random fashion. The data at each sensor is generated by simulation at the location of that sensor by means of *PetRos EiKon's* 3D Magnetic modeling functions.





The determinant at each point of the 4 eqn system for de-rotation



Transverse vs. Easting

Flight Path North

Gradient

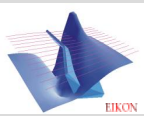
20.00
10.00
0.00
-10.00
-20.00

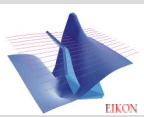
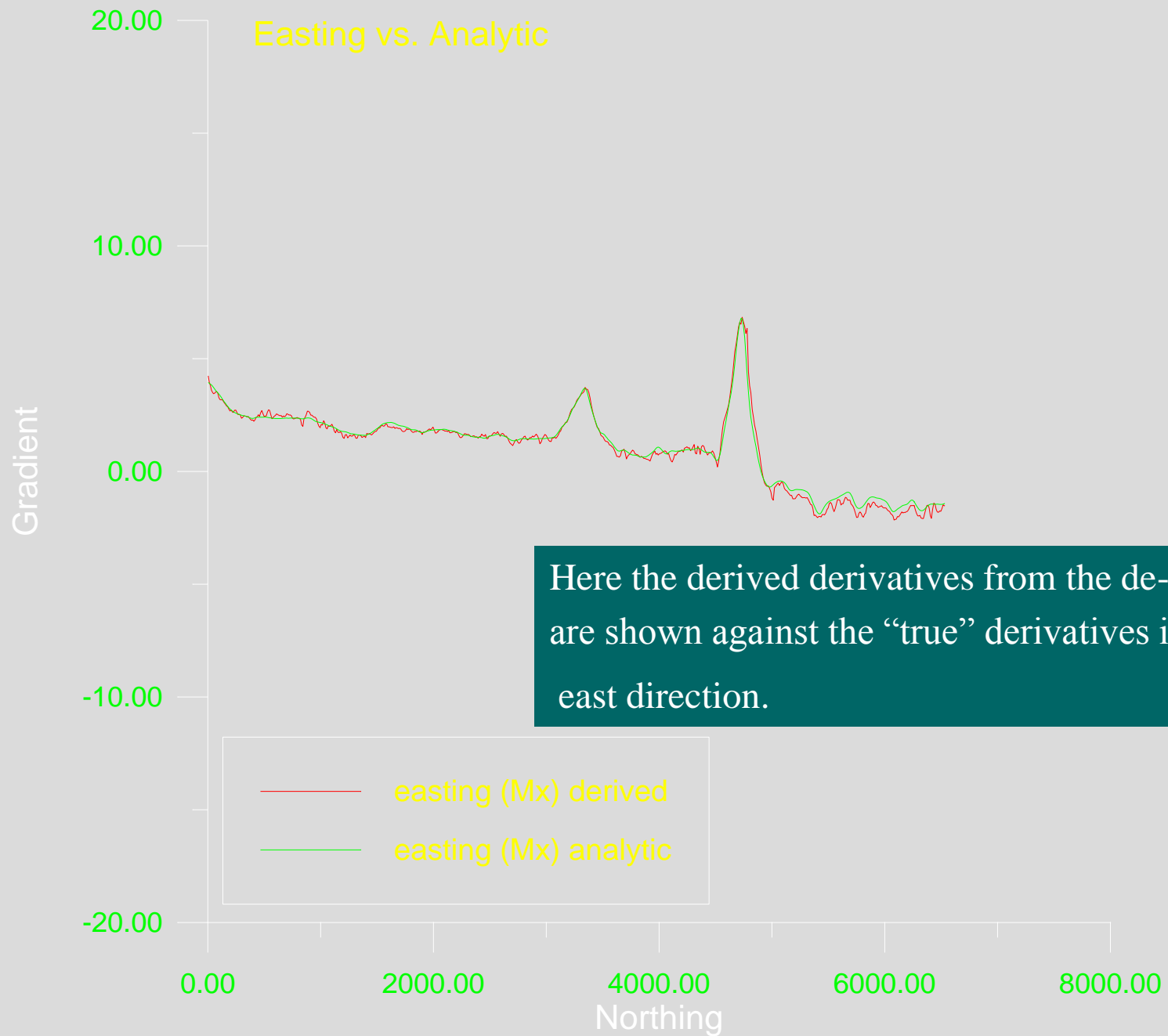
0.00 2000.00 4000.00 6000.00 8000.00

Northing

- transverse (-Mv)
- easting (Mx) derived

For the simulation, the flight path is generally North and here we show the simulated transverse “measured” derivative (blue) against the de-rotated derivative (red)





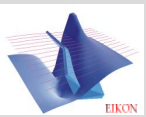
Synthetic (simulated) data example with true flight and orientation information

Single survey line (LINE10) decimated to approximately 620 stations

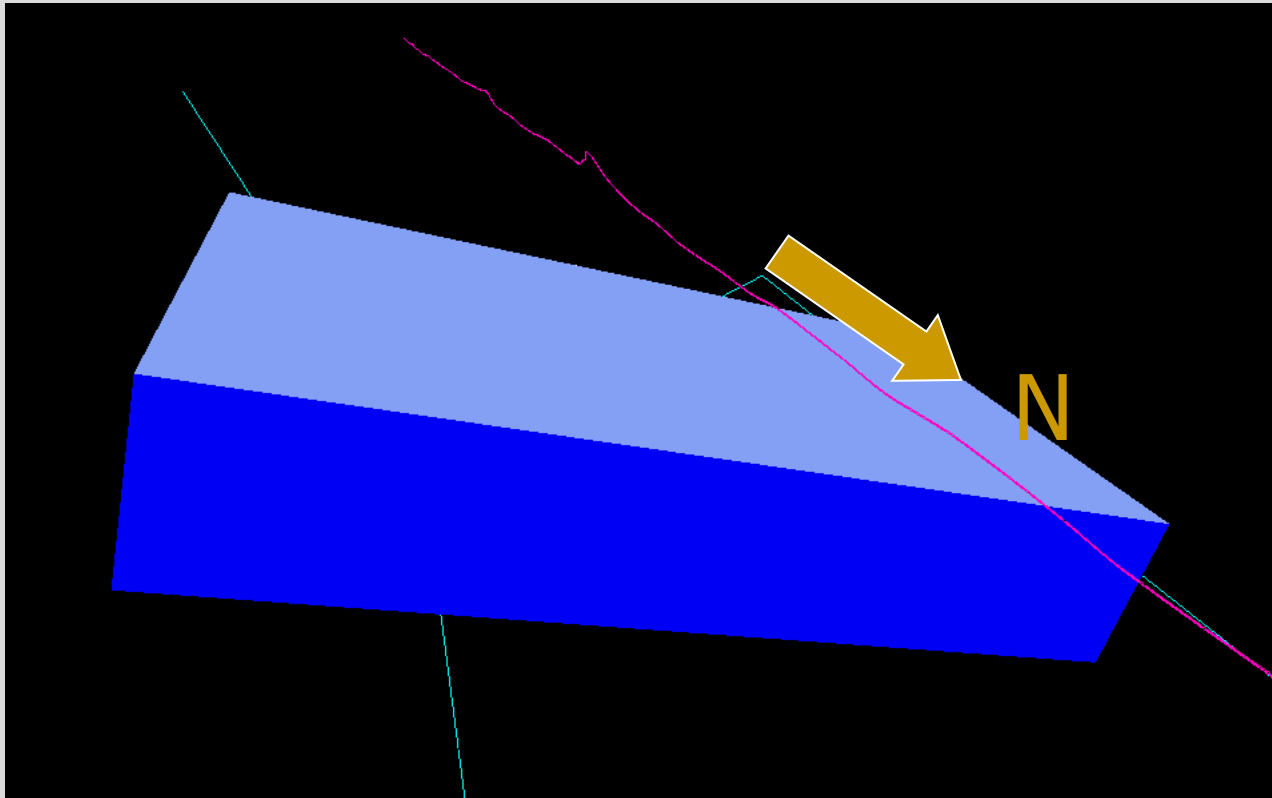
Simulated data is generated on three profiles which follow the trajectories of the three sensors based on given flight path and orientation information

> (x,y,z), pitch, roll, heading are from real data

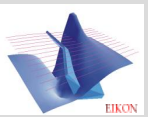
For this example, we utilize actual data locations and sensor orientations but use synthetic data to check processing technique



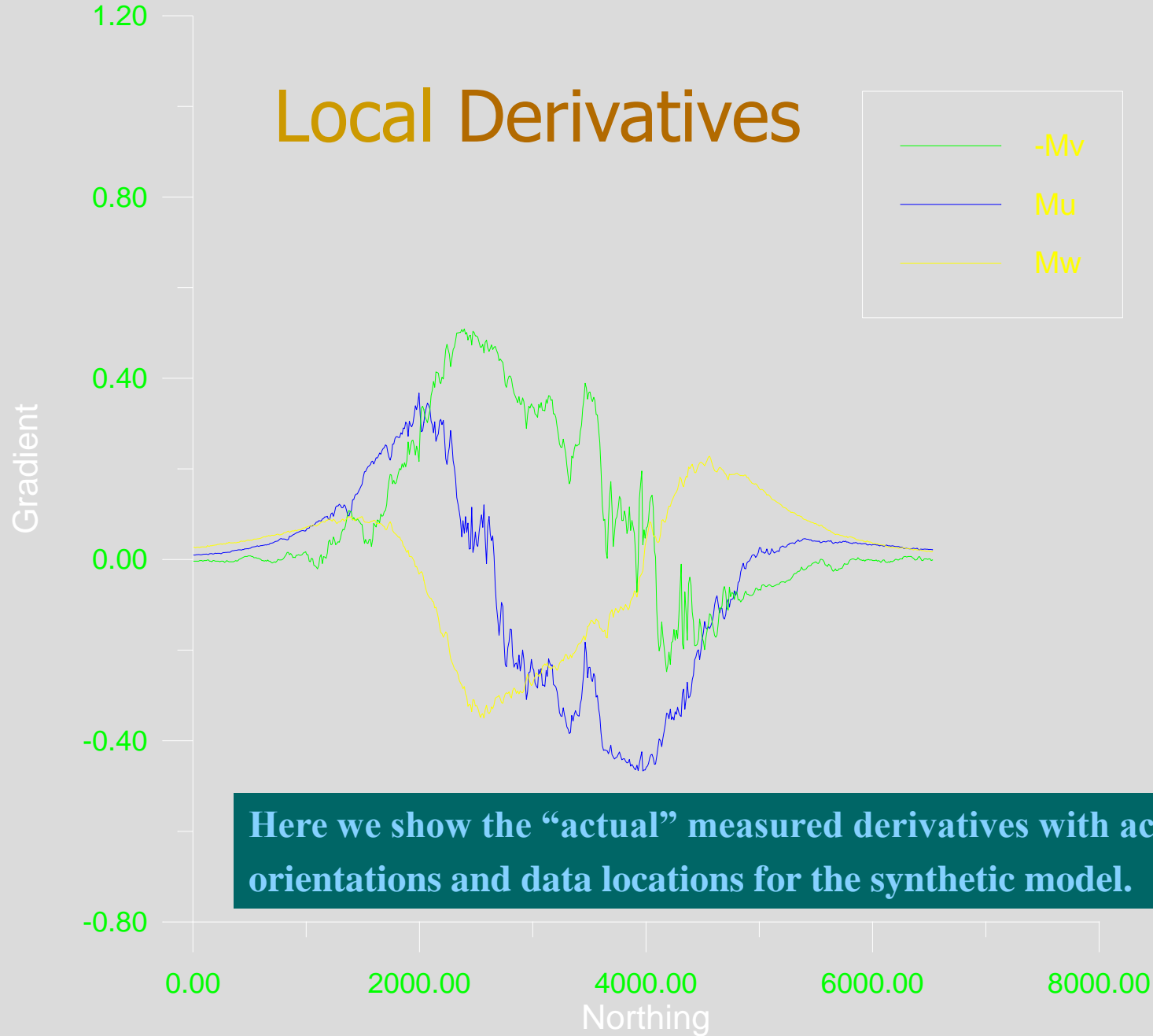
Magnetic prism model



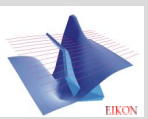
Here we show the synthetic model and the actual flight path with elevation for synthesis.



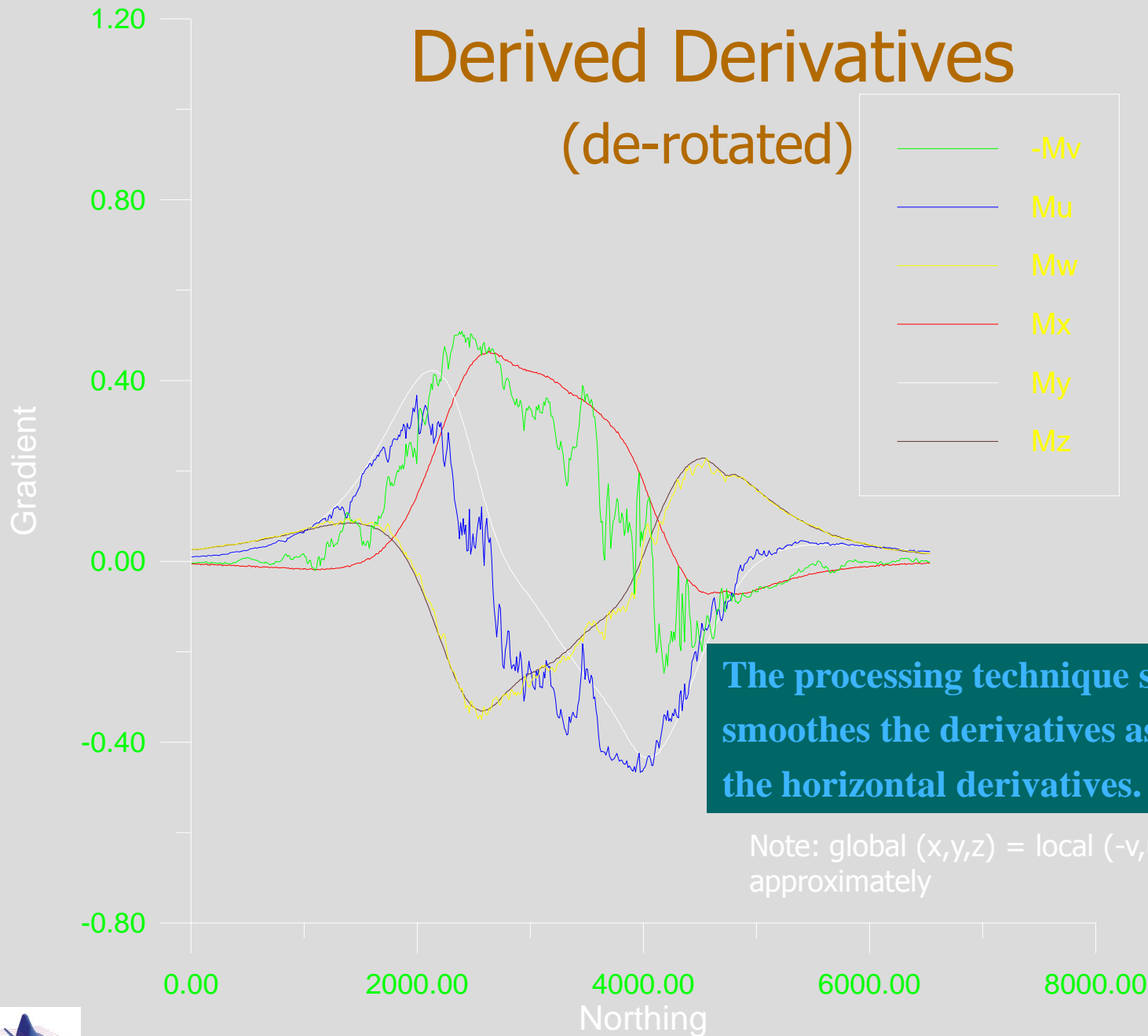
Local Derivatives



Here we show the “actual” measured derivatives with actual sensor orientations and data locations for the synthetic model.

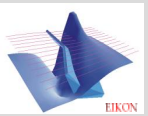


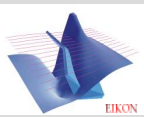
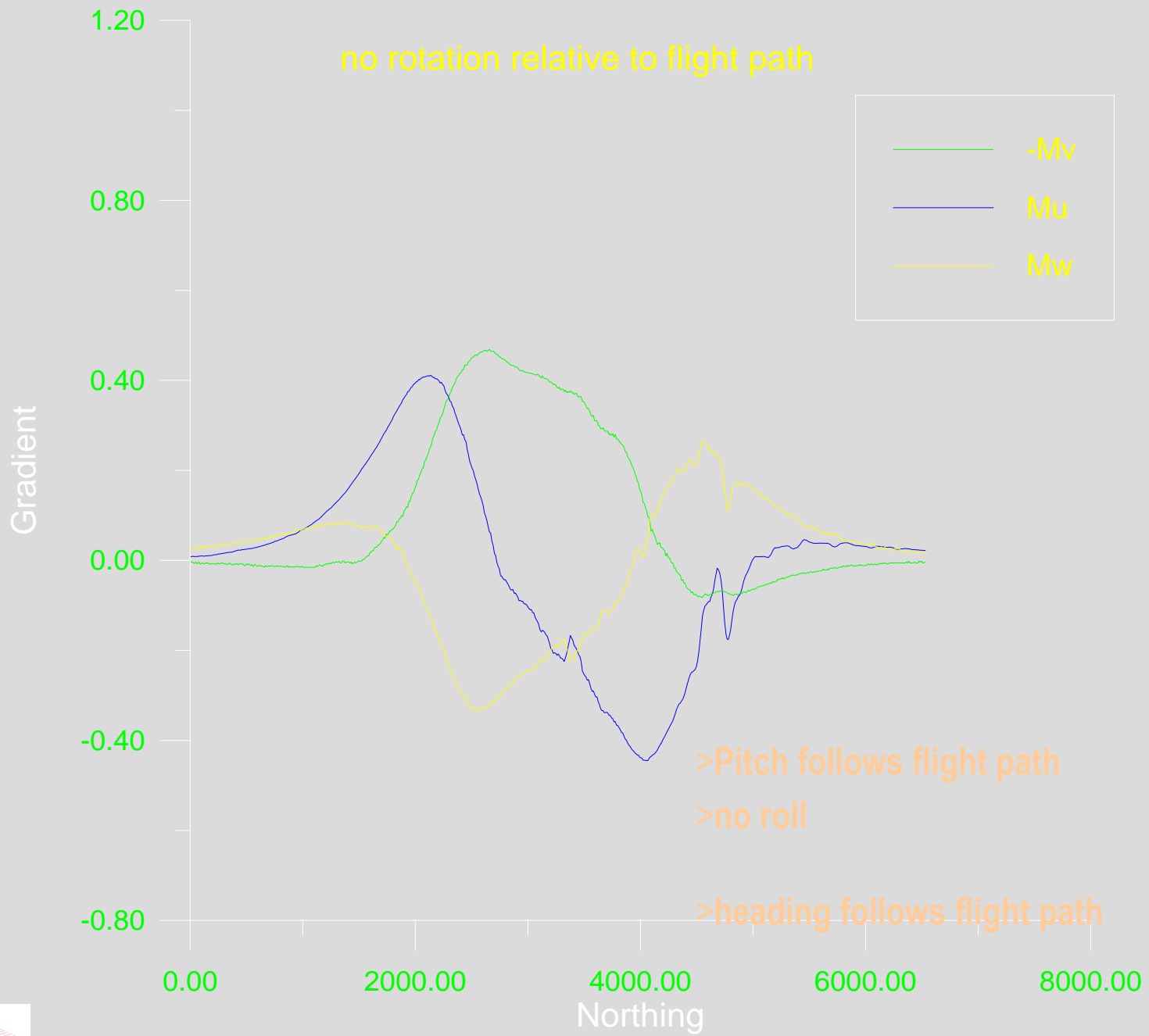
Derived Derivatives (de-rotated)

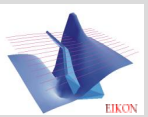
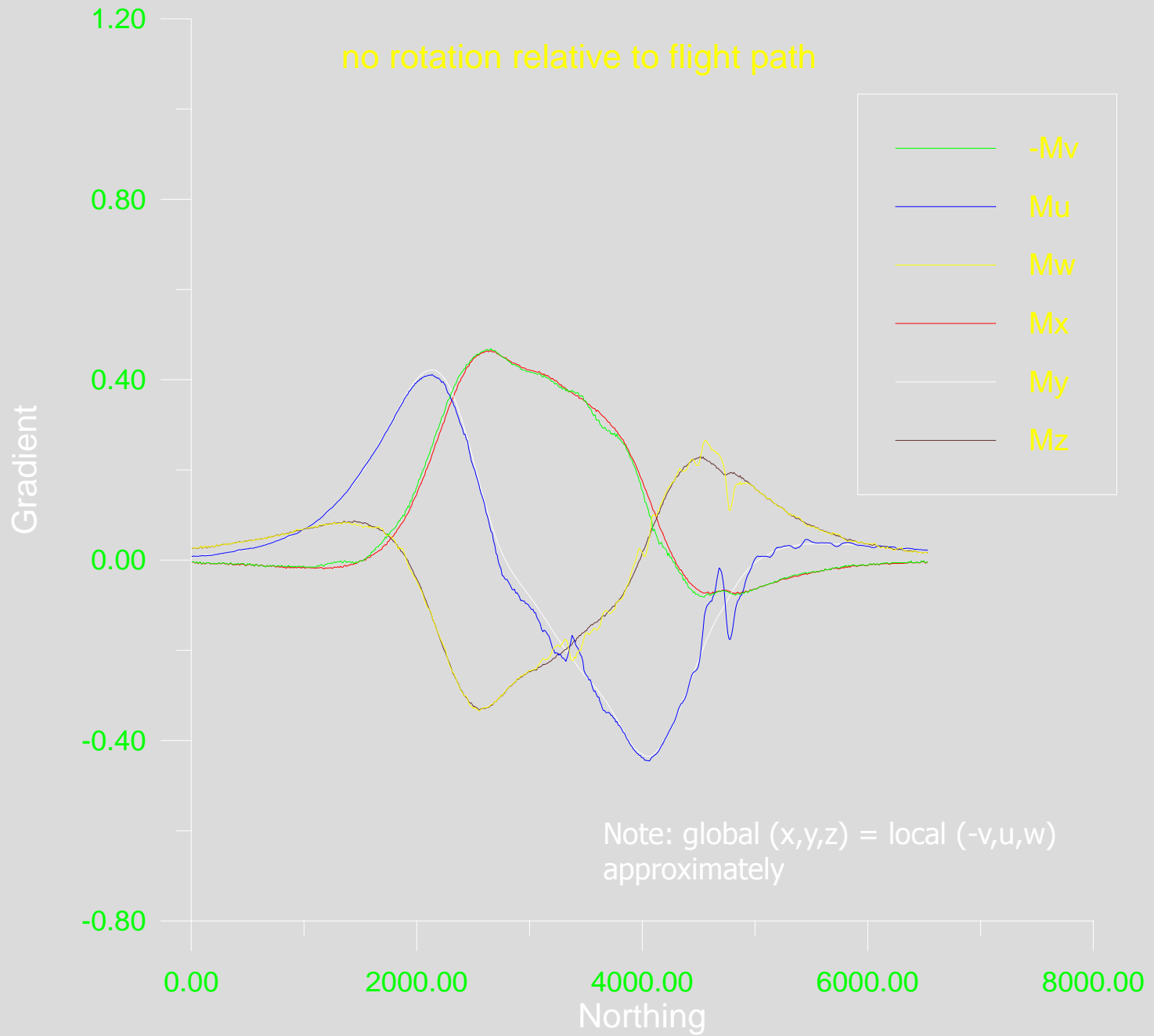


The processing technique significantly smooths the derivatives as well as un-mixing the horizontal derivatives.

Note: global (x,y,z) = local (-v,u,w) approximately







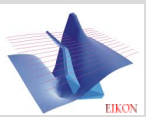
Gradient derivation from real data

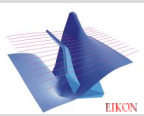
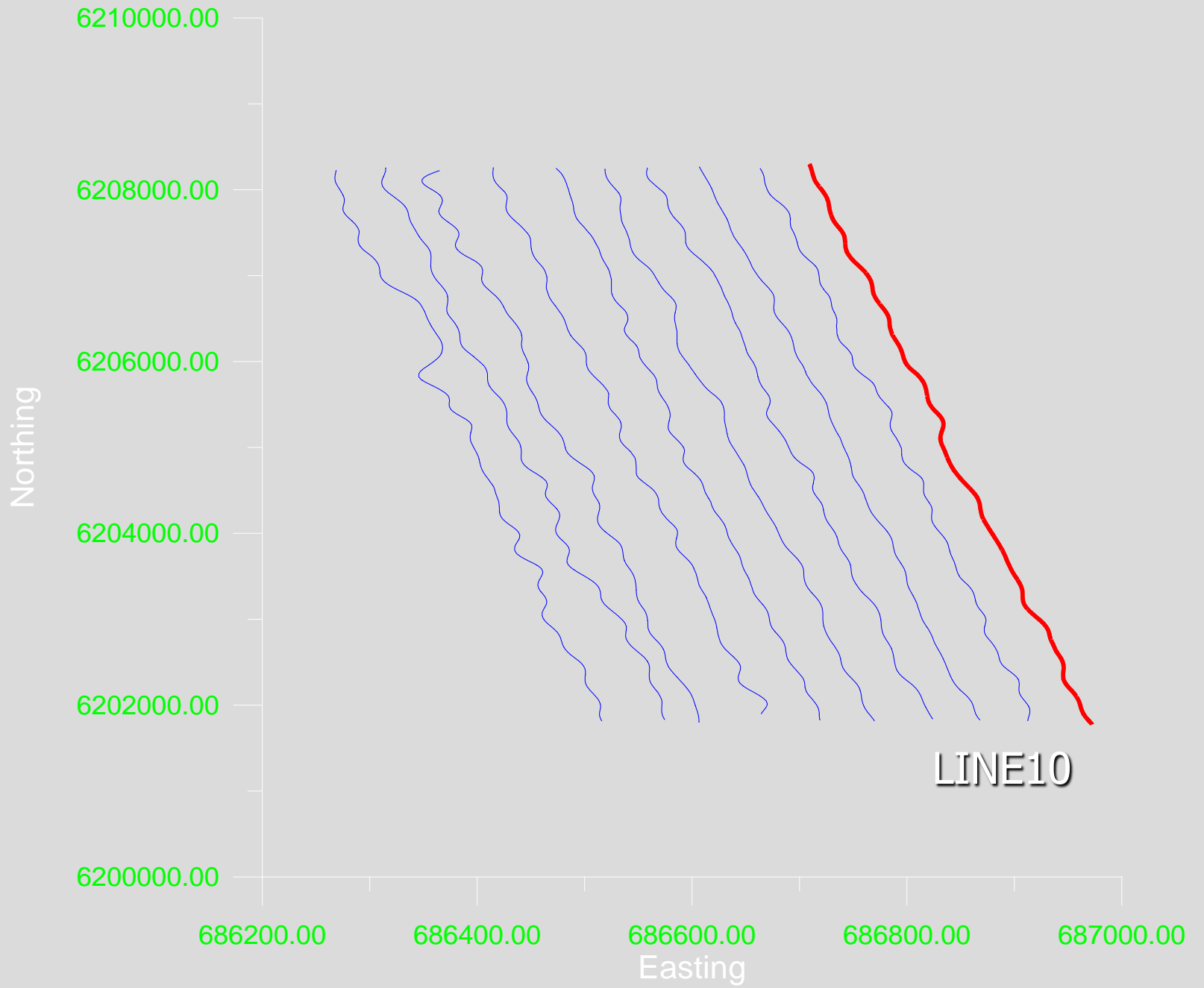
Measured “corrected” data

– (LINE10, LINE70)

Processed to derivatives based on given flight path and orientation information

Here we examine several effects of examining or using derivatives without correct processing





Declination: 22.86 degrees East of North

Gradient

2.00
0.00
-2.00
-4.00
-6.00
-8.00

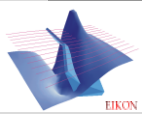
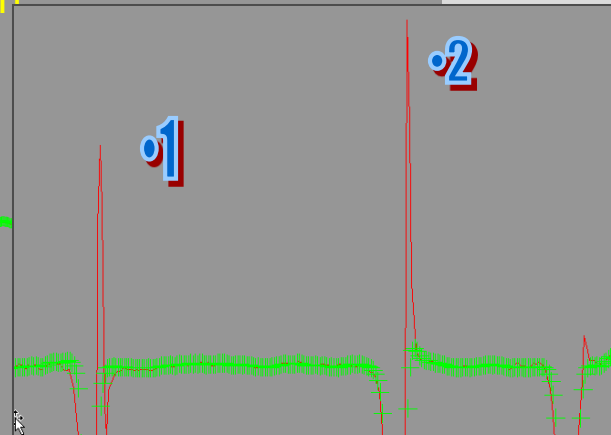
620000.00 620200.00 620400.00 620600.00 620800.00 621000.00

Northing

East Derivative



The transverse or easting derivative exhibits a negative-positive response across 2 anomalies which are not seen in the un-processed derivatives

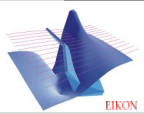
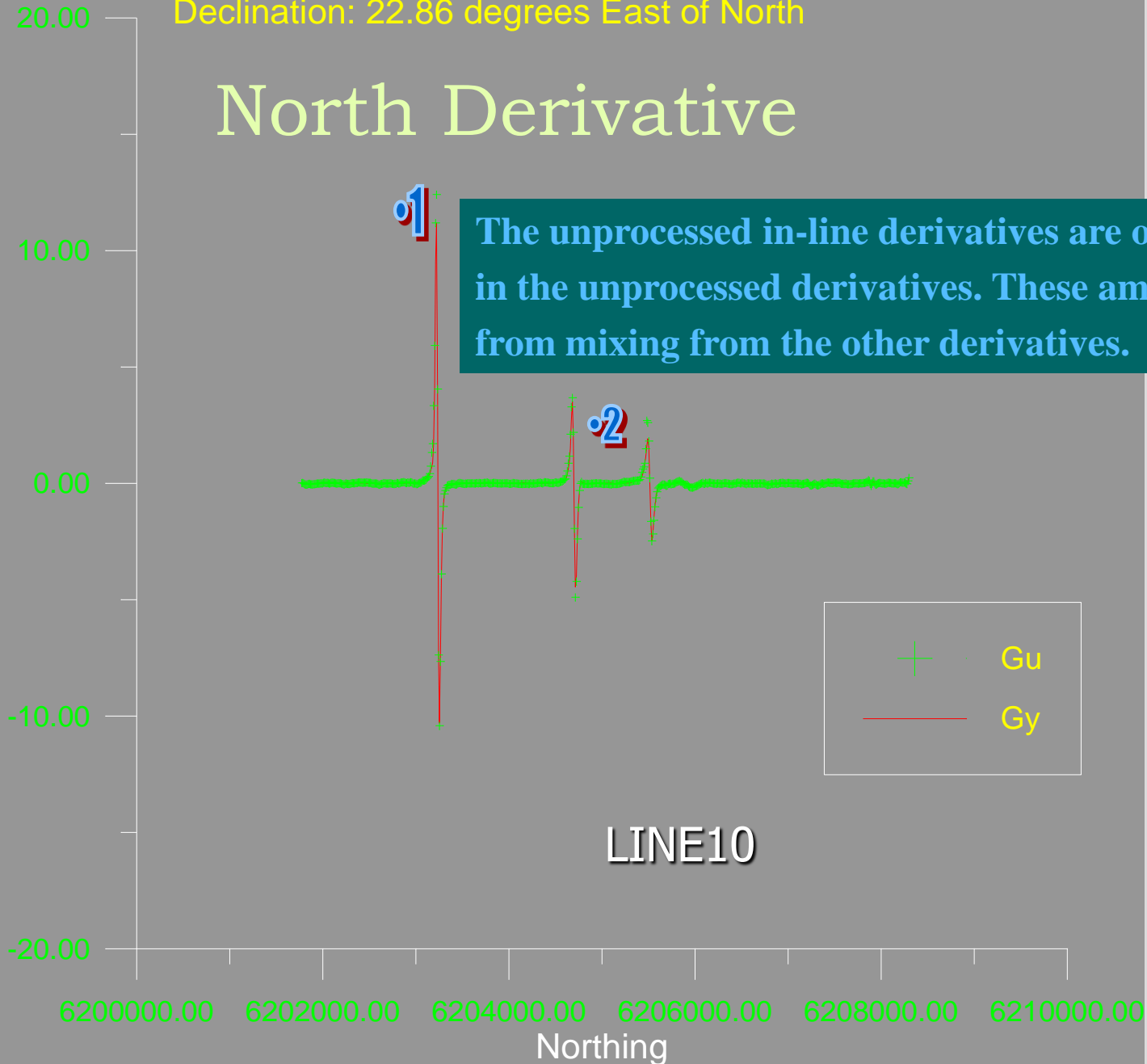


Declination: 22.86 degrees East of North

North Derivative

The unprocessed in-line derivatives are overestimated in the unprocessed derivatives. These amplitudes come from mixing from the other derivatives.

Gradient



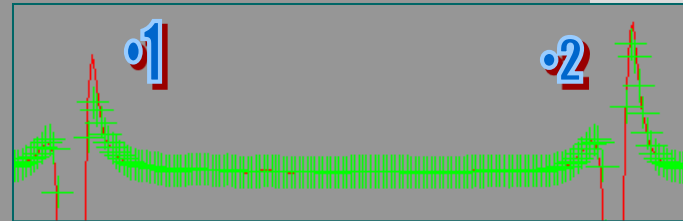
Declination: 22.86 degrees East of North

Gradient

10.00
0.00
-10.00
-20.00
-30.00

620000.00 620200.00 620400.00 620600.00 620800.00 621000.00

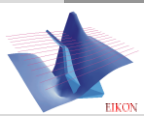
Northing



Vertical Derivative



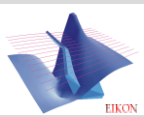
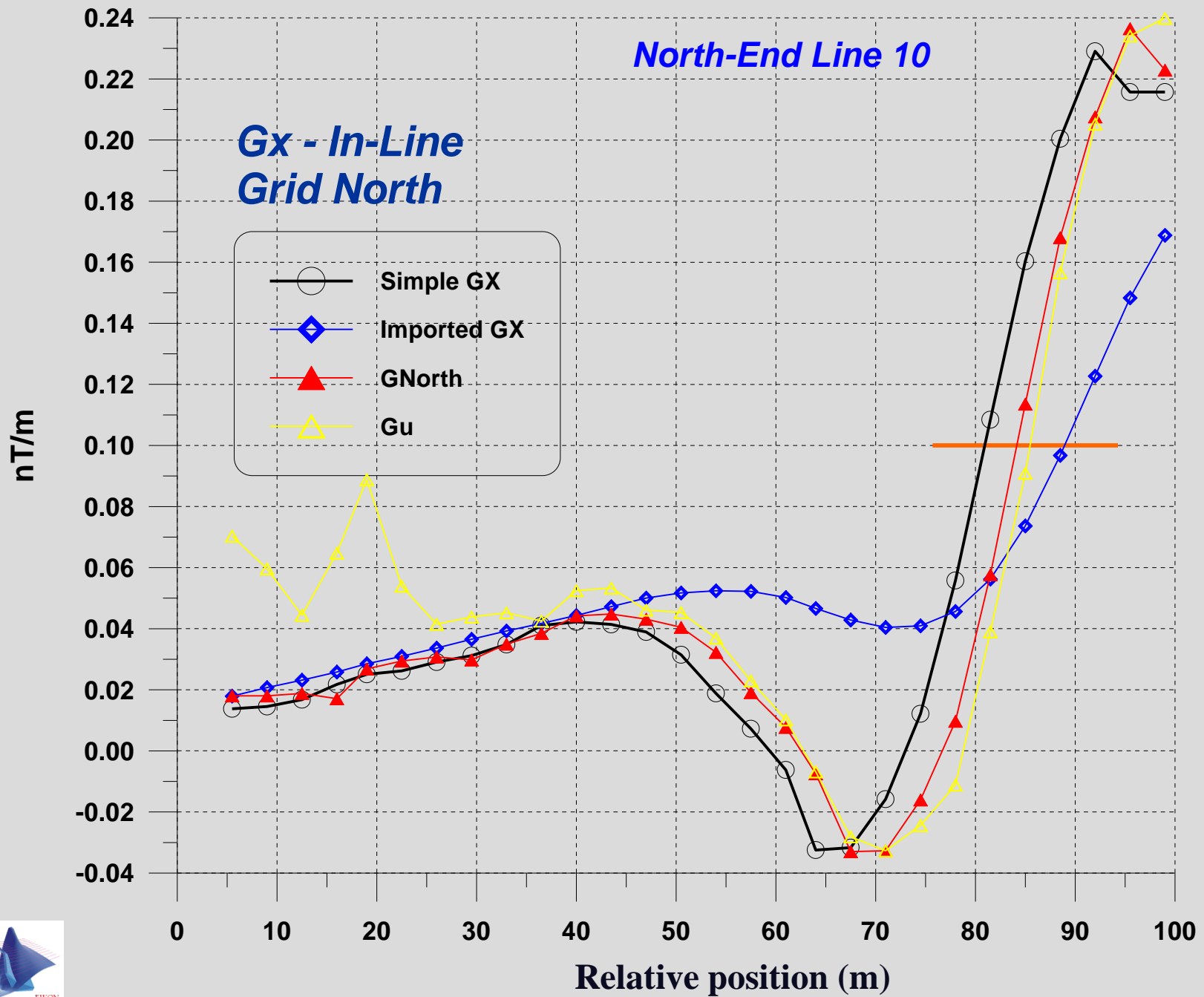
LINE10

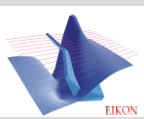
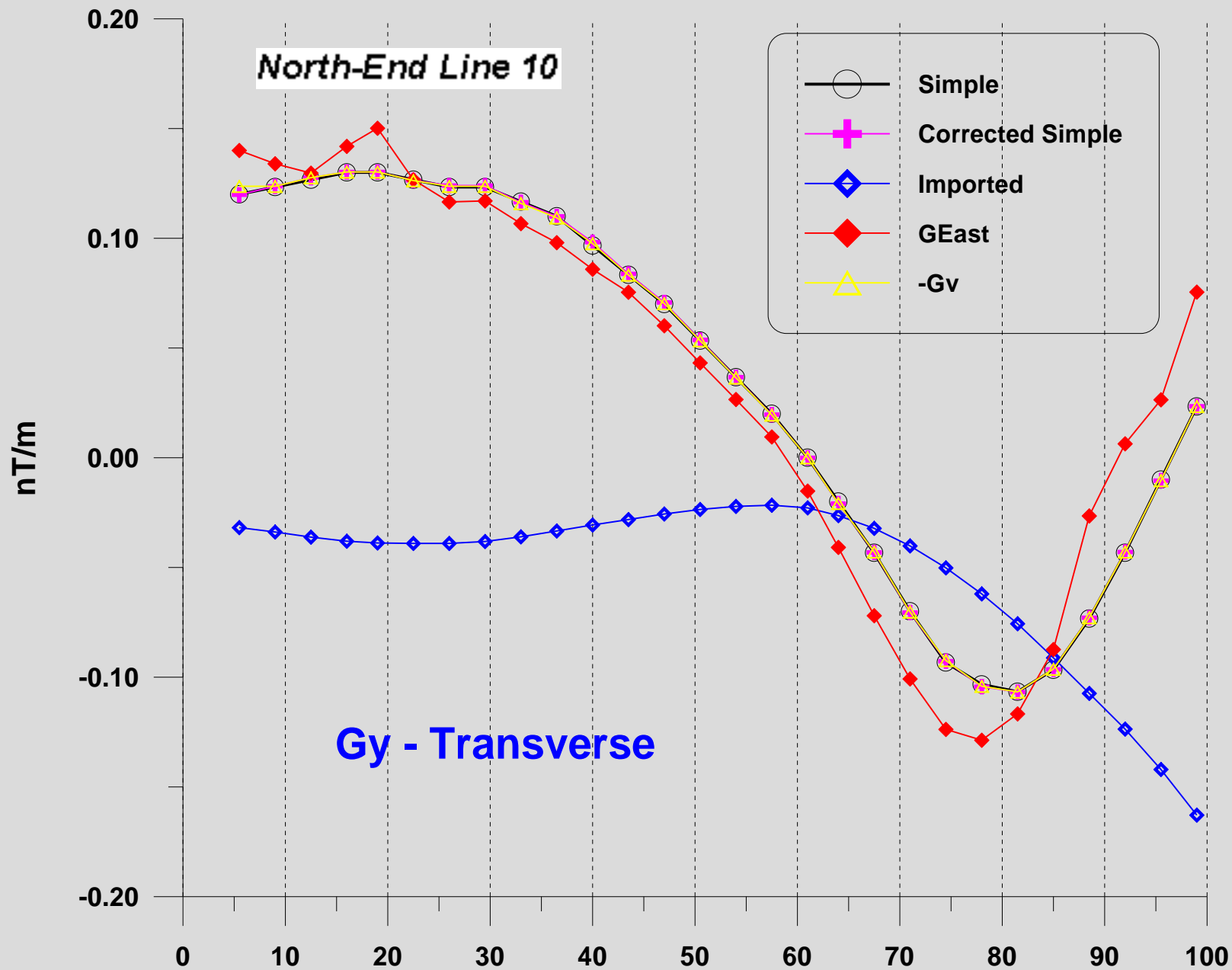


Correct Positioning of Derivatives

•In the following figures;

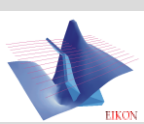
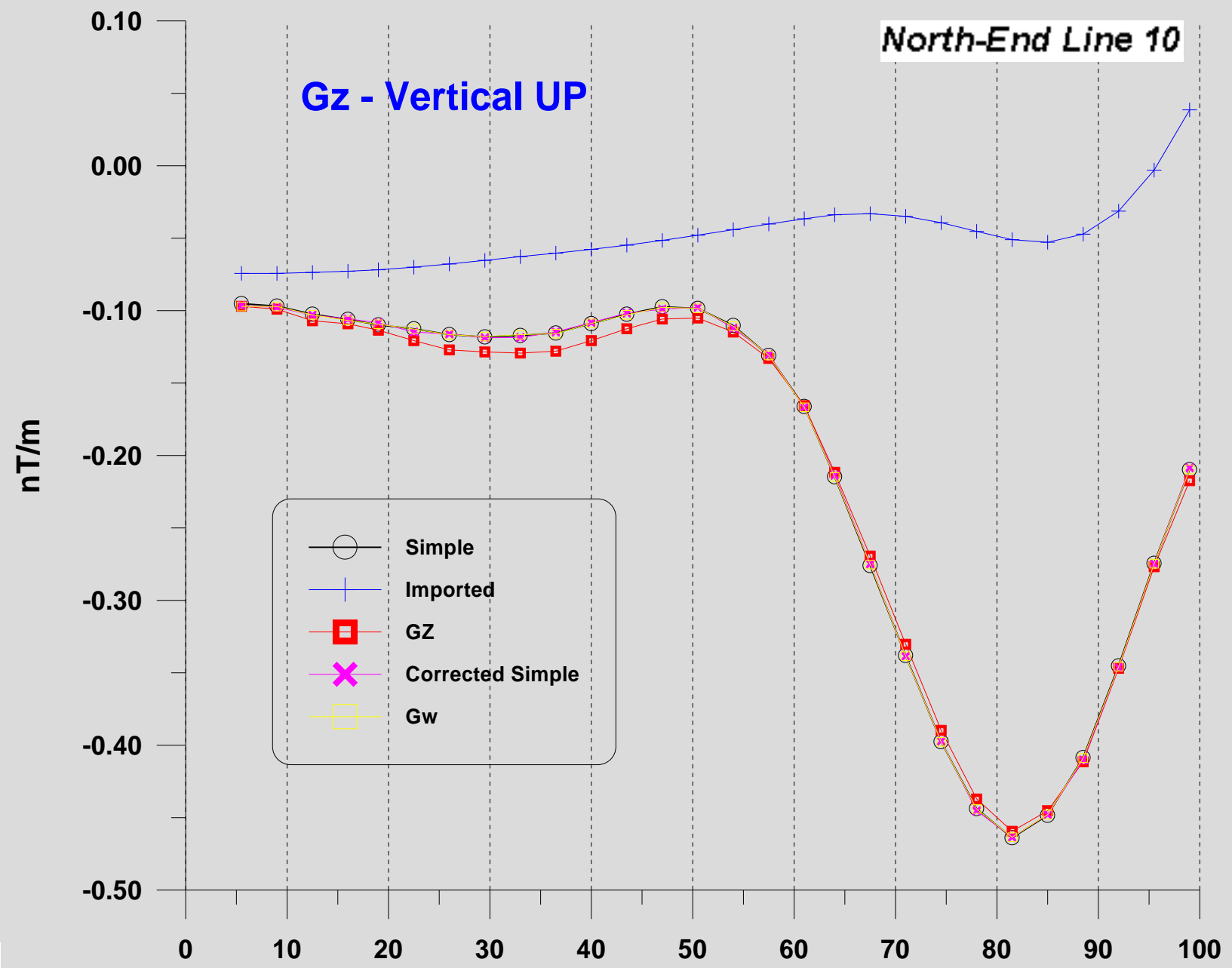
- Δ - Instantaneous collected derivative prior to processing
- **O** - Simple rotation of in-line based on measured derivatives being in a fixed frame
- \diamond - Alternative processing
- Δ - derivative in global (grid) frame – de-rotated derivatives
- + - Alternative simple rotation technique





North-End Line 10

Gz - Vertical UP



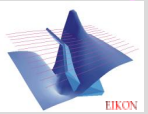
• Example 2

Fourier Transform Processing for Derivatives

1) All processed signals from the TMI data are generated by some transformation from the TMI derivatives (e.g. vertical derivatives, analytic signal, reduction-to-the pole). Traditionally, these derivatives are derived from Fourier transform of the TMI generally by FFT techniques.

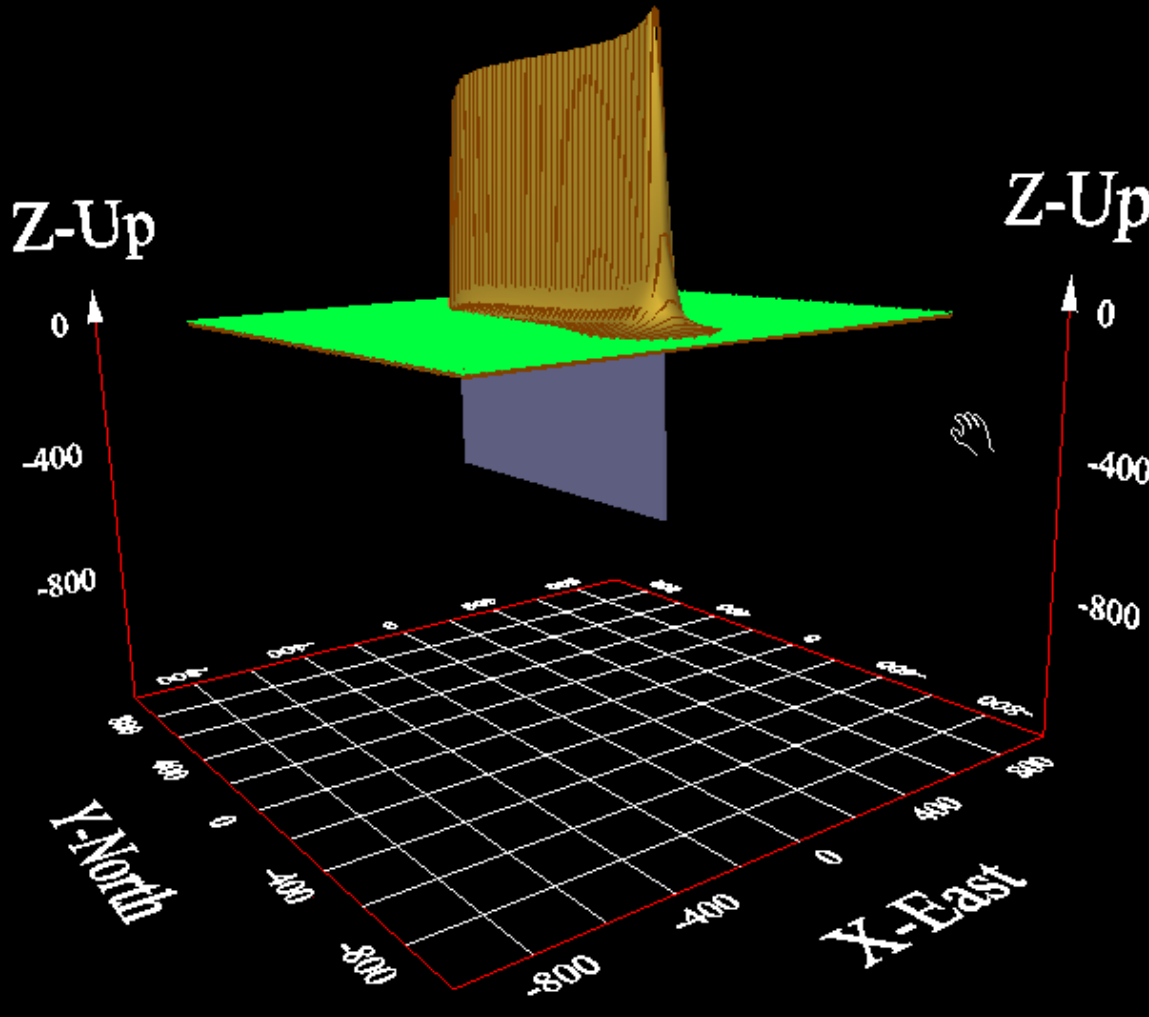
2) *PetRos EiKon* has extended its simulation algorithms to synthesize the derivatives of the TMI. These derivatives are not calculated by difference techniques from data at different positions on the grid but rather by extending the quasi-analytic formulation to calculate instantaneous (by position) derivatives at each data point. This is done by extending the Integral Equation formulation for the components to spatial derivatives of the components. The extensions are available for the normal Born calculation (magnetization parallel to Earth's field) and for non-linear effects (e.g. magnetic channelling, de-magnetization, interacting structures, remanent magnetization, etc).

Combining these two techniques in our newest software release allows the investigation of many aspects of traditional TMI processing. We will examine a couple of aspects here.



Fourier Transform Processing for Derivatives

- The Model

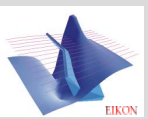


The model for this study is a thin dyke, 1km in length, striking N-S.

The inclination of the Earth's field is 75 degrees and the declination is 20 degrees East of North. The intensity is 52,500 nT.

The survey area is 1575 x 1575m, profile lines are 25m apart and data points are 25m apart.

For this example, the synthetic data is determined on a regular grid to illustrate various features.



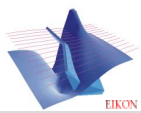
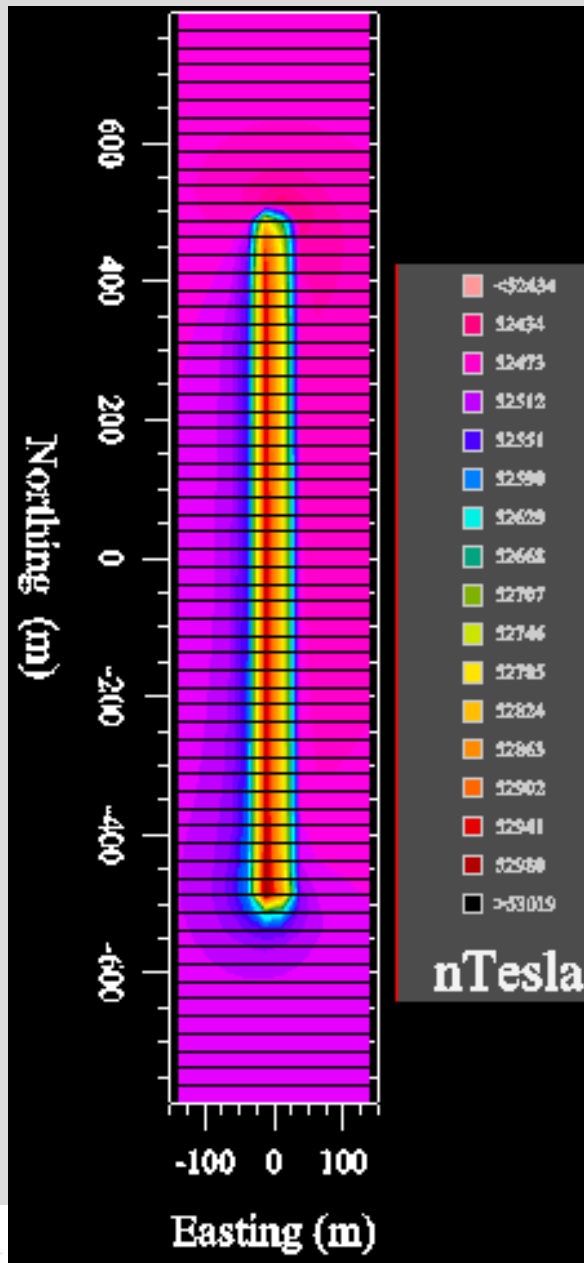
Fourier Transform Processing for Derivatives - TMI

TMI Simulated

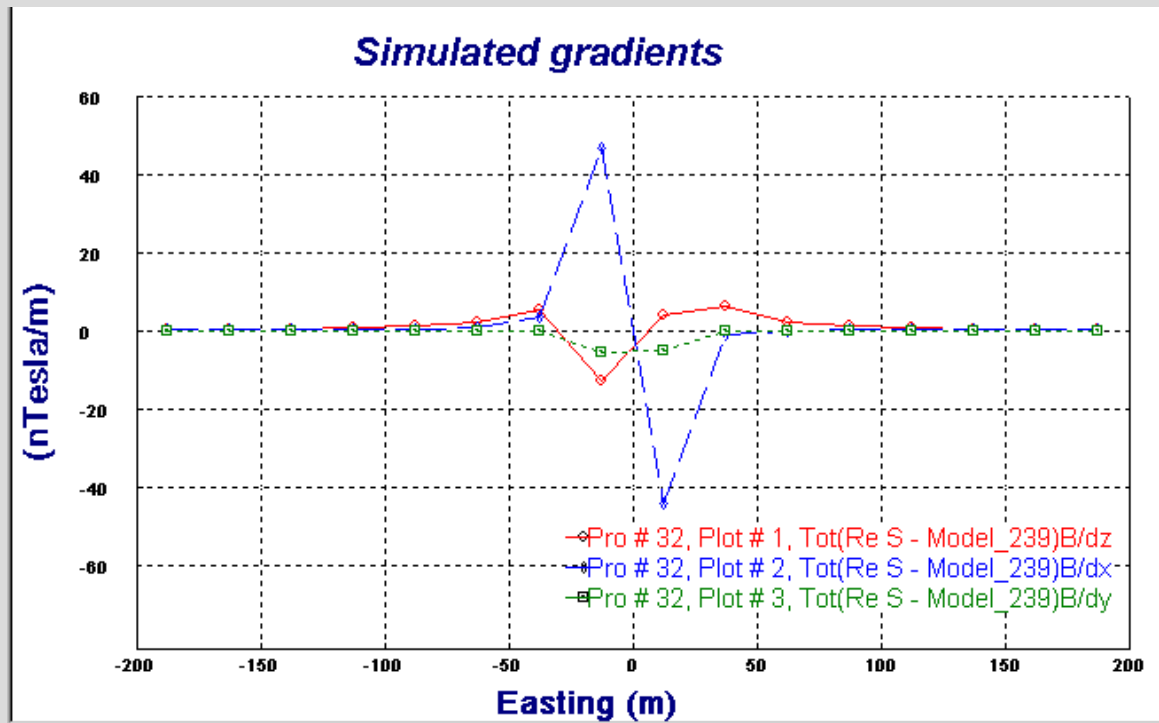
Here we display the simulated TMI. The interpolated data is on a grid which is exactly that of the simulated data points.

Note the asymmetry in the TMI response due to the inclination and declination of the earth's field.

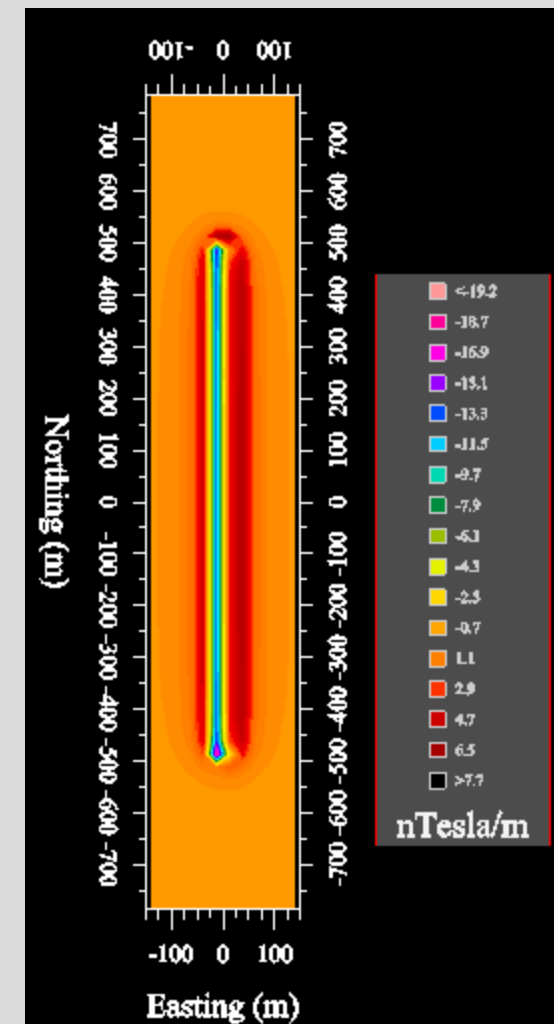
The dyke comes to within 5m of the earth's surface and has a depth extent of 500m.



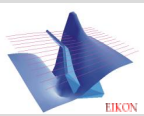
Fourier Transform Processing for Derivatives - Derivatives



Here we display the simulated derivatives across a central line of the anomaly. And to the right, the contoured vertical gradients on the original data grid.



Simulated Vertical Gradients



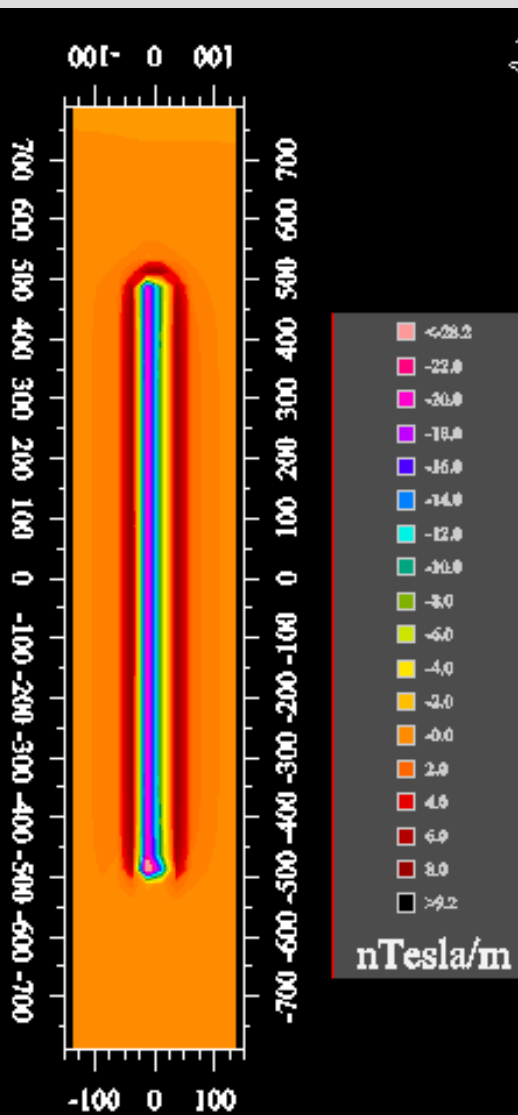
Fourier Transform Processing for Derivatives

- Derivatives

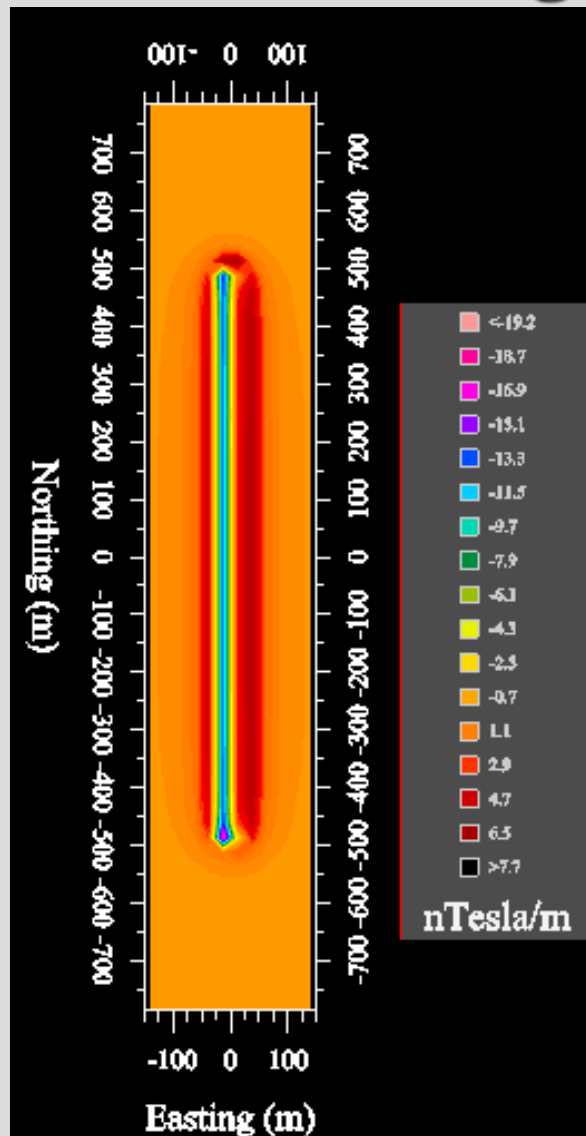
This is an extremely interesting Figure as it demonstrates that the Technique of deriving derivatives By FFT is more-or-less justified.

For those unfamiliar with the proof Of such techniques, the original Mathematical justification for the Fourier transforming for the Derivatives is not fully proven. Also, Using an FFT for the Fourier transform is somewhat contrary Is proper mathematics.

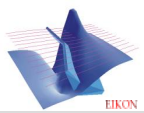
However, as can be seen by comparison of the figures. The FFT technique does overestimate the Vertical gradient and imparts variation which is also not actually in the derivative.



Vertical Gradient by FFT



Vertical Gradient by Simulation



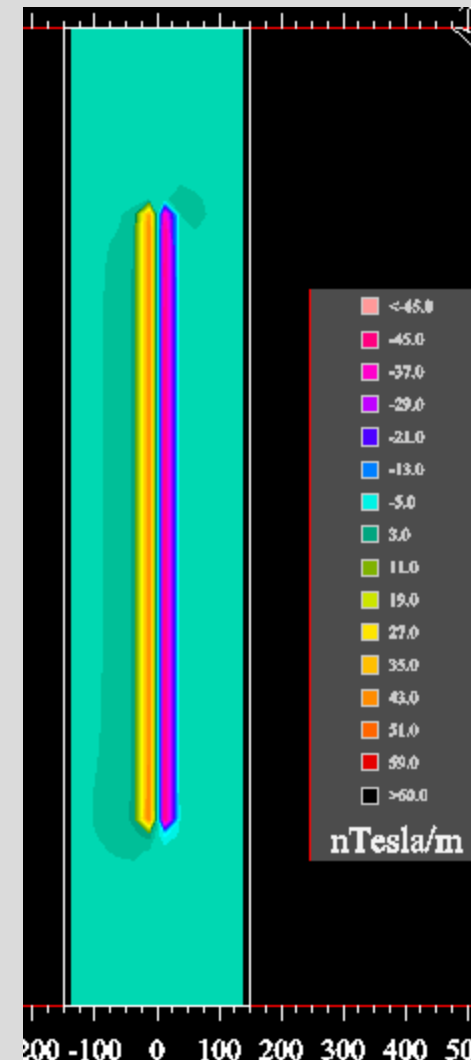
Fourier Transform Processing for Derivatives

- Derivatives

What is more interesting is the comparison of the in-line horizontal derivatives (d/dx).

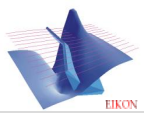
Note that the in-derivatives by FFT are significantly underestimate. Compare the figures and that of the x-y plot of this derivatives (2 pages back).

Note the rippling caused by the FFT as expected.



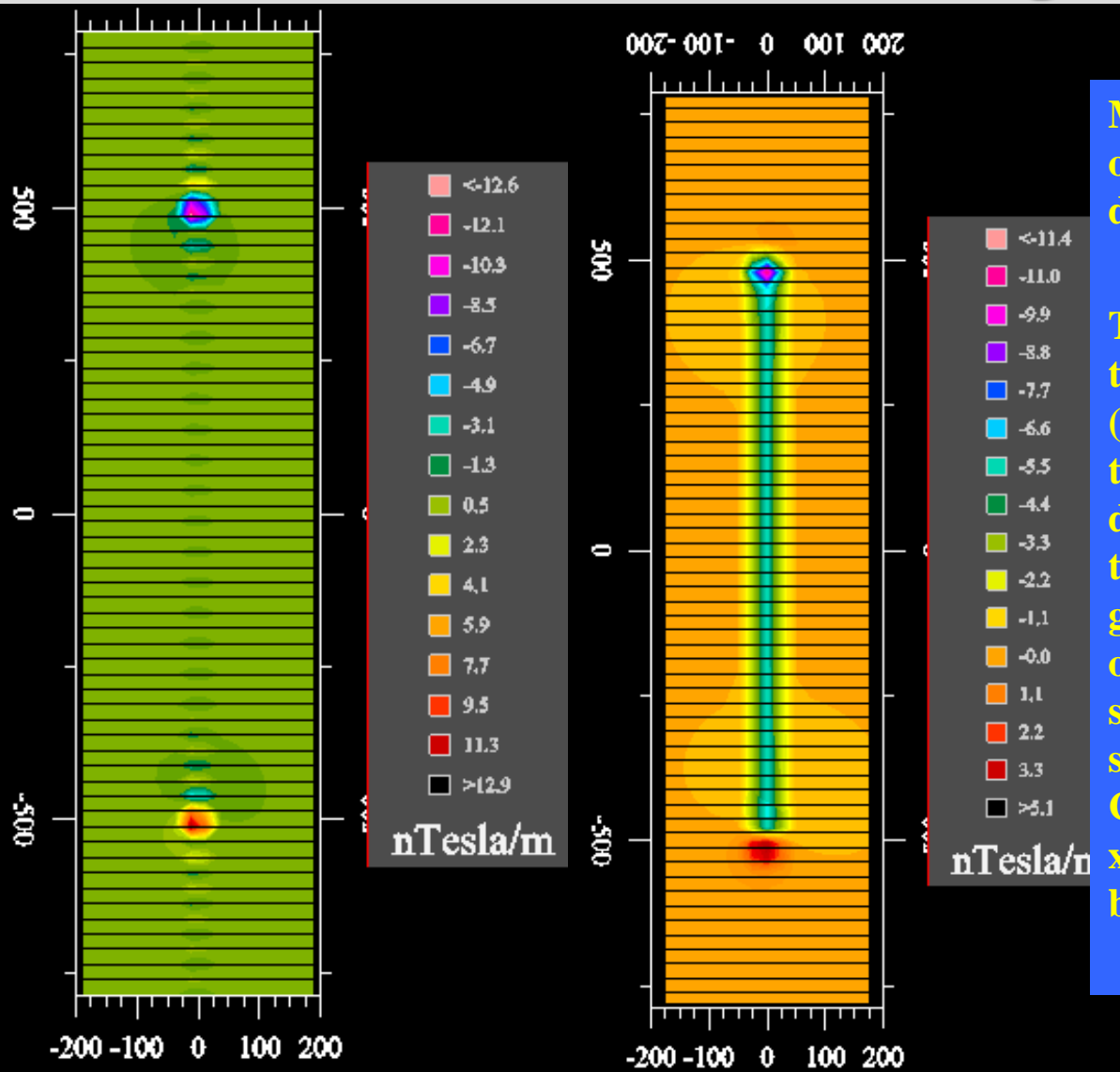
In-Line Gradient by FFT

In-Line Gradient by Simulation



Fourier Transform Processing for Derivatives

- Derivatives



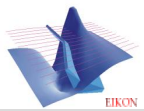
More disturbing is the comparison of the cross-line horizontal derivatives (d/dy).

The FFT technique overestimates the derivatives at the ends (particularly in the south). Shows the rippling effects caused by the discrete samples in the FFT and the taper window at the edge of the grid. And cannot discriminate the object in the center but rather shows the object as 2 dipole like structures only at the ends.

Compare the figures and that of the x-y plot of this derivatives (3 pages back).

Cross-Line Gradient by FFT

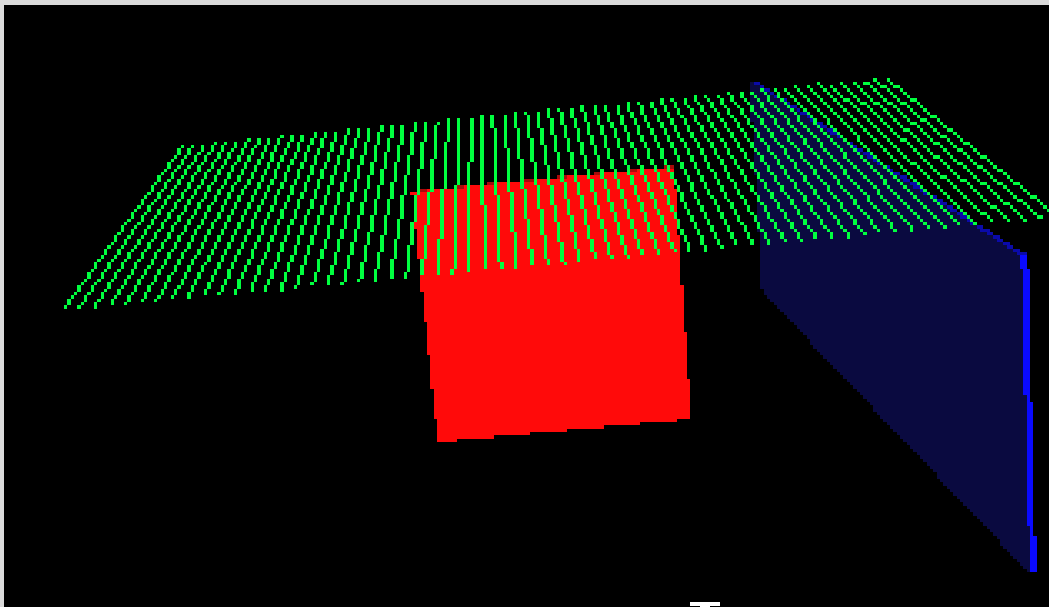
Cross-Line Gradient by Simulation



Gridding (Interpolation) with Derivative Information

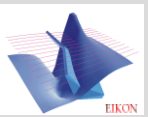
• Example 3

Another interesting aspect of measuring gradients is the consideration that the derivatives may be used to enhance (increase resolution) of the data grids. This would be accomplished by using interpolation techniques which would utilize the measured gradients to increase the density of the interpolated output grid from the profile data.

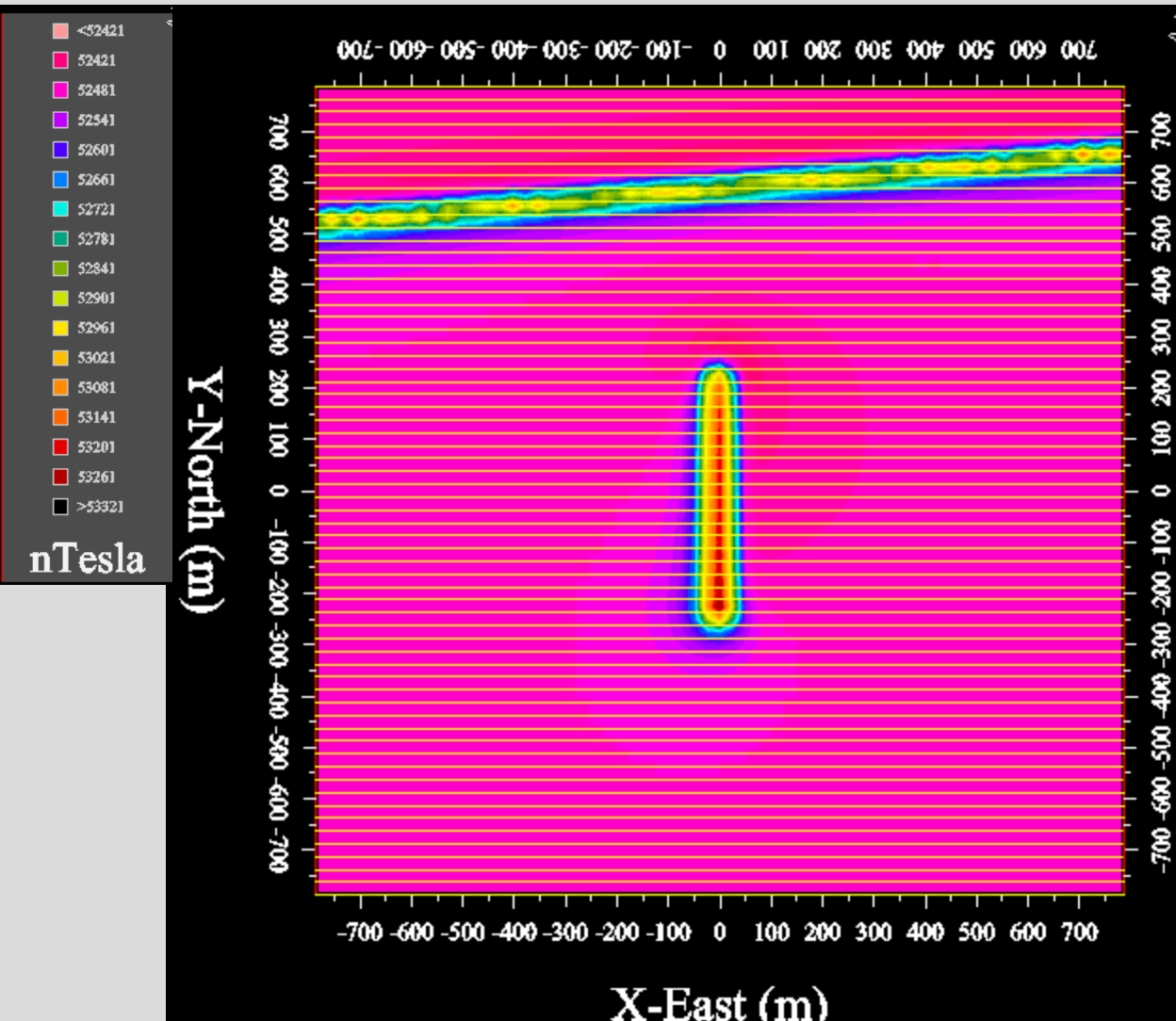


We can also investigate this aspects with the use of our new software tools.

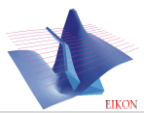
Here we will consider a new model beneath the previous data grid. The fault is reduced in size and another object (larger than the grid) is introduced which is sub-parallel to the data lines (i.e. almost parallel).



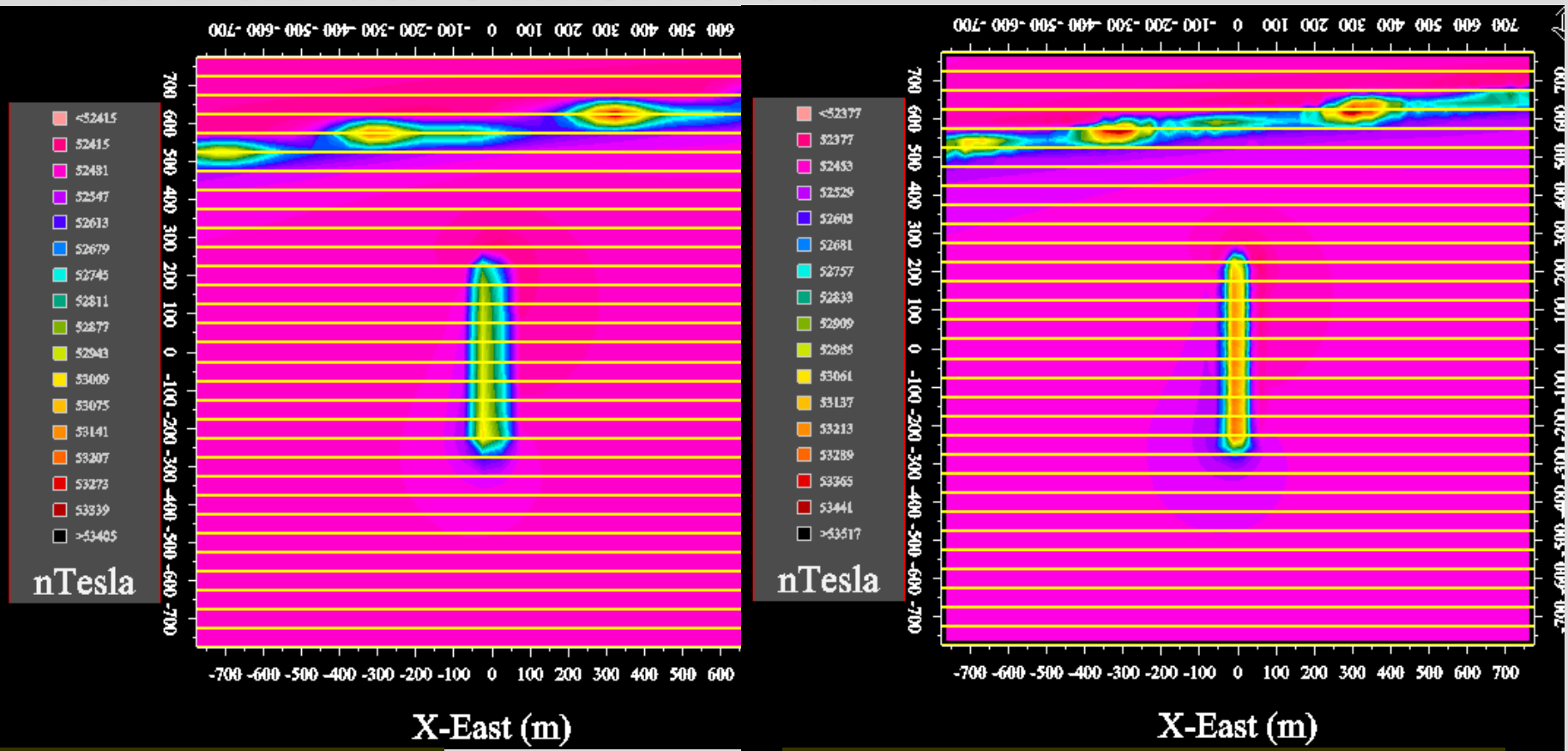
Gridding (Interpolation) with Derivative Information



For this grid, both objects are clearly delineated by the 25x25m data sampling.



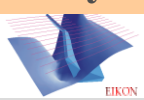
Gridding (Interpolation) with Derivative Information



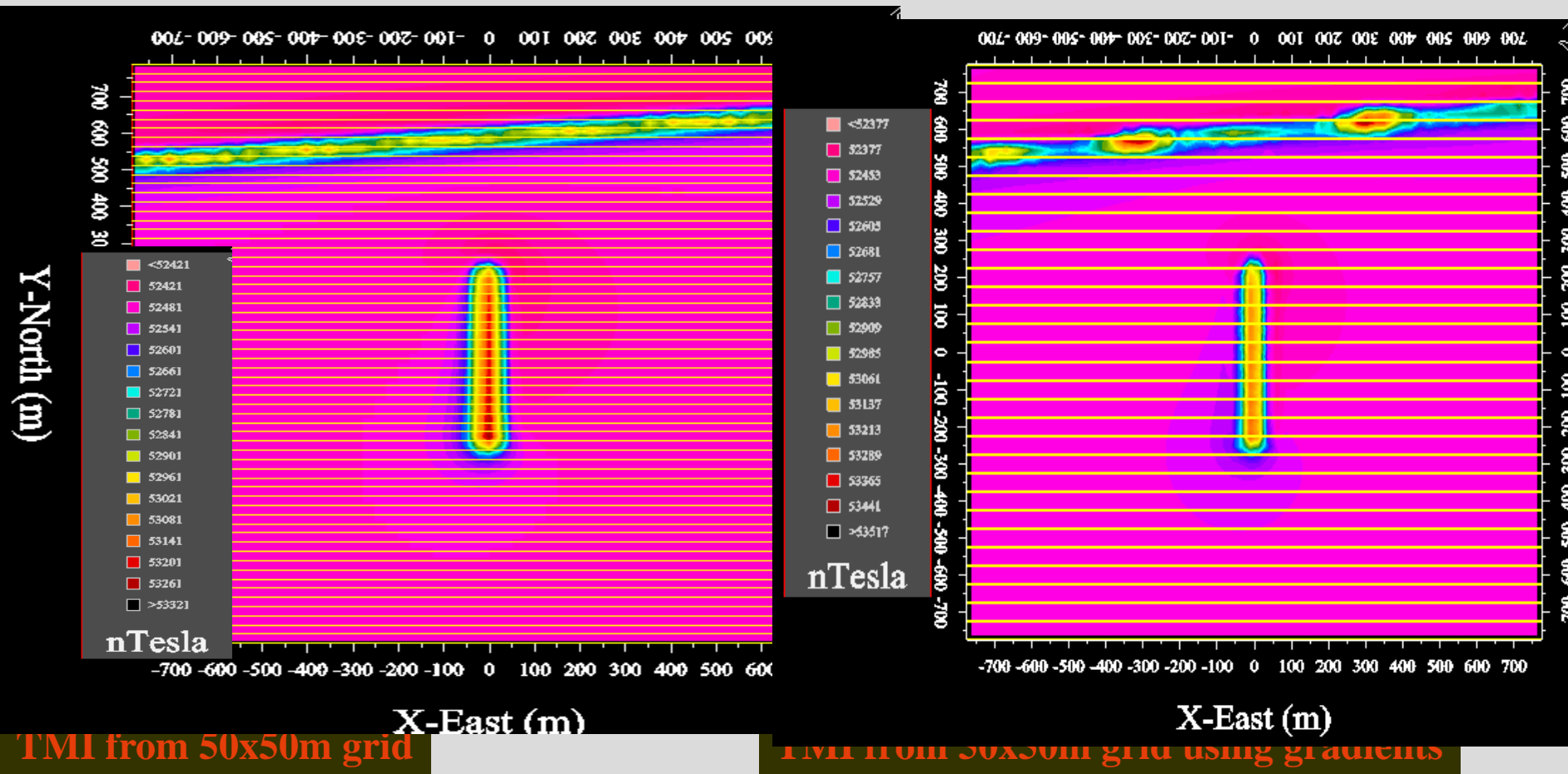
TMI from 50x50m grid

TMI from 50x50m grid using gradients

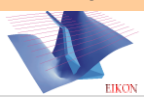
The use of the gradients does improve the resolution both of the E-W structure but almost more clearly outlines the N-S structure. Data sampling in these grids are not precisely as in the more dense grid (previous figure). However, the gradient gridding very closely reproduces the high density “data”.

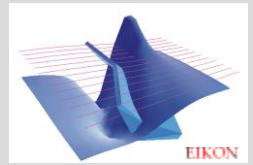


Gridding (Interpolation) with Derivative Information



The use of the gradients does improve the resolution both of the E-W structure but almost more clearly outlines the N-S structure. Data sampling in these grids are not precisely as in the more dense grid (previous figure). However, the gradient gridding very closely reproduces the high density “data”.





3 Examples of *PetRos EiKon* research as pertains to the use of measured TMI derivatives

Conclusions:

3 pertinent examples of the use of magnetic gradients showing different aspects of our research project.