# Processing gradients of magnetic data utilizing an equivalent technique

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**Summary**

We experimented with synthetic ground data to generate derivatives of total magnetic field data utilizing an equivalent layer with a distribution of magnetic susceptibility. The depth of the equivalent layer plays an important role in producing the derivatives. Placing the equivalent layer too shallow will introduce high frequency content that will severely contaminate the generated derivatives. We discretized the equivalent layer into a set of rectangular cells whose horizontal dimensions are proportional to the inline-spacing and across-line distance. Based on our experiment with synthetic data, an equivalent layer placed at a depth equal to the line-spacing will provide reasonably good derivatives. We compared the derivatives generated with equivalent source technique and compared with derivatives utilizing an FFT technique. In the synthetic cases that we studied, the derivatives generated with an equivalent layer at an appropriate depth are closer to the true (simulated) derivatives than the FFT derivatives.

**Introduction**

The equivalent source technique has been studied widely in the literature (Dampney (1969), Emila (1973), Mendonca and Silva (1994, 1995), Li (2001)). Dampney (1969) utilized an equivalent source of discrete masses on a plane to project the measurements of a Bouguer anomaly onto a regularly gridded horizontal plane. Emila (1973) used an equivalent source of lines of dipoles to compute derivative fields, upward- and downward-continuation fields along a single profile. Mendonca and Silva (1994, 1995) developed methodologies to interpolate potential field data utilizing an equivalent layer. Li (2001) processed gravity gradiometer data using an equivalent layer of density varying laterally.

In previous work (Jia *et al*, 2005), we utilized equivalent source techniques to derive gradients of gravity data. This approach was reasonably straightforward and produced excellent results. When we utilized our original algorithms with magnetic data, the results were sometimes unsatisfactory. In this article, we studied the issues regarding generating the entire gradient vector of total magnetic field (TMF) utilizing an equivalent source technique. The issue of generating all three gradients seems fundamental since if the 3 gradients can be generated accurately then most other processing techniques follow. We first outline the procedure for constructing the equivalent susceptibility models. We then apply a forward technique to compute derivatives of the TMF from the equivalent model. The derivatives thus obtained will be analyzed and compared with 2D FFT derivatives. We utilized synthetic data to demonstrate that the derivatives generated with equivalent layer at appropriate depth can be of superior quality than the derivatives calculated by FFT.

**Method**

Our goal of the equivalent source construction is to find a fictitious source that can then be modeled to produce derivatives of the TMF. We invert for a layer of magnetic susceptibility that varies laterally and is placed at some distance below the observation stations. We also experimented with utilization of multiple layers of susceptibility as our equivalent source. However, the results thus obtained are not significantly different from a single layer case and thus we focus our attention on single layer in this article. The susceptibility layer is discretized into a set of contiguous cells, each of which has constant susceptibility. The grid is orientated parallel to the average survey line azimuth and thus each cell has one dimension along this azimuth (inline) and one dimension perpendicular (cross-line). The inline cell dimension is half of the average inline data sampling interval while the cross-line dimension is half of the average line spacing. As pointed out in Dampney (1969), and Mendonca and Silva (1995), the depth of the equivalent source should not be smaller than 2.5 times the spacing between data points, otherwise some form of aliasing may occur. In a survey where the spacing between profiles is substantially greater than the spacing between data stations, the use of a shallow equivalent source can produce artificial features in the computed derivatives of TMF in the direction perpendicular to the profiles. We will utilize synthetic data to show that the depth of the equivalent source plays a very important role in generating derivatives, particularly in the cross-line direction.

We divide a susceptibility layer beneath the survey area into $M$ rectangular cells each of which has constant but unknown susceptibility. The susceptibility values of the $M$ rectangular can be collected as a model vector,

$$\overline{\mathrm{K}} = (k_1, k_2, \cdots, k_M)^T .$$

Assume that the TMF data are collected at $N$ data points, and let the measured data be expressed as

$$\overline{d} = (d_1, d_2, \cdots, d_M) .$$

Assuming that Born approximation is utilized as forward simulation (induced magnetization parallel to the earth's local field), the magnetic field produced by the equivalent source are related to the susceptibility values of cells by a linear equation system

$$G\overline{\mathrm{K}} = \overline{d}$$

where $G$ is the coefficient matrix whose $(i, j)$-element $g_{ij}$ specifies the contribution of *j*-th cell with unit susceptibility to the *i*-th datum. The measure of misfit is defined as

$$\phi_d = \left\| W_d (d - \hat{d}) \right\|^2 ,$$

where $W_d = diag(1/\sigma_1, 1/\sigma_2, \cdots, 1/\sigma_M)$, $\sigma_i$ is the standard deviation of the *i*-th datum. The equivalent source layer can then be determined by solving the minimization problem

$$\phi = \phi_d + \mu \phi_m ,$$

where $\mu$ is a regularization parameter and $\phi_m$ is a term that measures the smoothness of the susceptibility distribution of the equivalent layer. The optimal value of $\mu$ will produce a data misfit equal to its expected value from the noise statistics.

**Examples**

We built our synthetic models based on the survey geometry and model as utilized by Mendonca and Silva (1995). We utilized 9 survey lines each of length 800 m along the NS direction with a line spacing of 86m and a data sample every 8.6 m. The survey elevation is 1 m. We assume that the earth's field is inclined at 45 degrees to the horizontal and oriented 45 degrees to the north and has strength of 52500 nT as in Mendonca and Silva paper. We inserted a cube 100 m on a side that has magnetic susceptibility of 0.05 (SI). The equivalent layer is chosen to be 20 percent larger than the data area. To compare the derivatives obtained with equivalent source technique against 2D FFT derivatives, we interpolated the data onto a variety of regular grids. Here we will show the FFT derivatives derived with a grid cell dimension 12 m along the inline direction and 45 m along the cross-lines direction. We then utilized a standard FFT technique to compute the gradients. We determined the equivalent layer at 20 m and 80 m respectively. Once, the equivalent source was determined, the derivatives of the equivalent layer were computed at the grid vertices utilized in the FFT. The true, FFT and the derivatives from equivalent layers at the two depths are displayed in Figure 1. For the sake of limited space here, we only illustrate the results for the cross-line derivatives as this is the most difficult derivative to derive. It is seen that the equivalent layer at depth of 80 m gives better results than the FFT while the equivalent layer at depth of 20 m introduces some high frequency content into the derivatives, demonstrating the importance of placing the equivalent layer at appropriate depth. Next, we added independent Gaussian noise of standard deviation 0.7 nT to the simulated total field data. We then determined the equivalent layer at depth 20 m and 80 m respectively. We also generated FFT derivatives using the noisy data. For comparison, we show FFT results with the grid settings as above. Again, the results demonstrate that also in case of noise contaminated data the equivalent layer at depth of 80 m produces more accurate results than by FFT for this difficult derivative. The equivalent layer at the depth of 20 m again introduces artificial high frequency content into the derivatives.

For the inline derivative, the FFT technique and equivalent source technique are virtually identical except for the very important issue that there are no edge effects in the equivalent source technique. There are only small differences between the 20m and 80m depth equivalent source solutions. In the case of noise, the equivalent source technique is superior in regions of stronger derivatives while the FFT is superior away from the anomaly. This is intuitive as the FFT technique spreads noise across the entire spectrum area while the equivalent source distributes the noise effects according to the actual spatial distribution of the noise. The noise was added uniformly throughout the survey region and not proportional to the data amplitudes. The conclusions for the vertical derivatives are similar to the conclusions for the cross line derivatives.

**Conclusions**

The distance between the observational surface and the equivalent layer or equivalent source (ES) is a very important factor in generating derivatives of total magnetic field. Placing the equivalent layer too close to the observation stations will introduce high frequency content that will severely contaminate the generated derivatives. We discretized the equivalent layer into a set of rectangular cells whose horizontal dimensions are proportional to the inline-spacing and cross-line distance. Based on our experiments with synthetic data, an equivalent layer placed at a depth equal to the line-spacing will provide superior TMF derivatives than by an FFT technique.

**References**

Dampney, C.N.G., 1969, The equivalent source technique, Geophysics, 34, 39-53.

Emilia, D.A., 1973, Equivalent sources used as analytical base for processing total magnetic field profiles, Geophysics, 38, 339-348.

Jia, R, Groom, R.W, Lo, B., 2005, On some issues regarding 3D-gravity inversion, 75th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts.

Li, Y., 2001, Processing gravity gradiometer data using an equivalent source technique, 71st Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts.

Mendonca, C.A., and Silva, J.B.C., 1994, The equivalent data concept applied to interpolation of potential field data, Geophysics, 59, 722-732.

Mendonca, C.A., and Silva, J.B.C., 1995, Interpolation of potential field data by equivalent layer and minimum curvature: a comparative analysis, Geophysics, 60, 399-407.

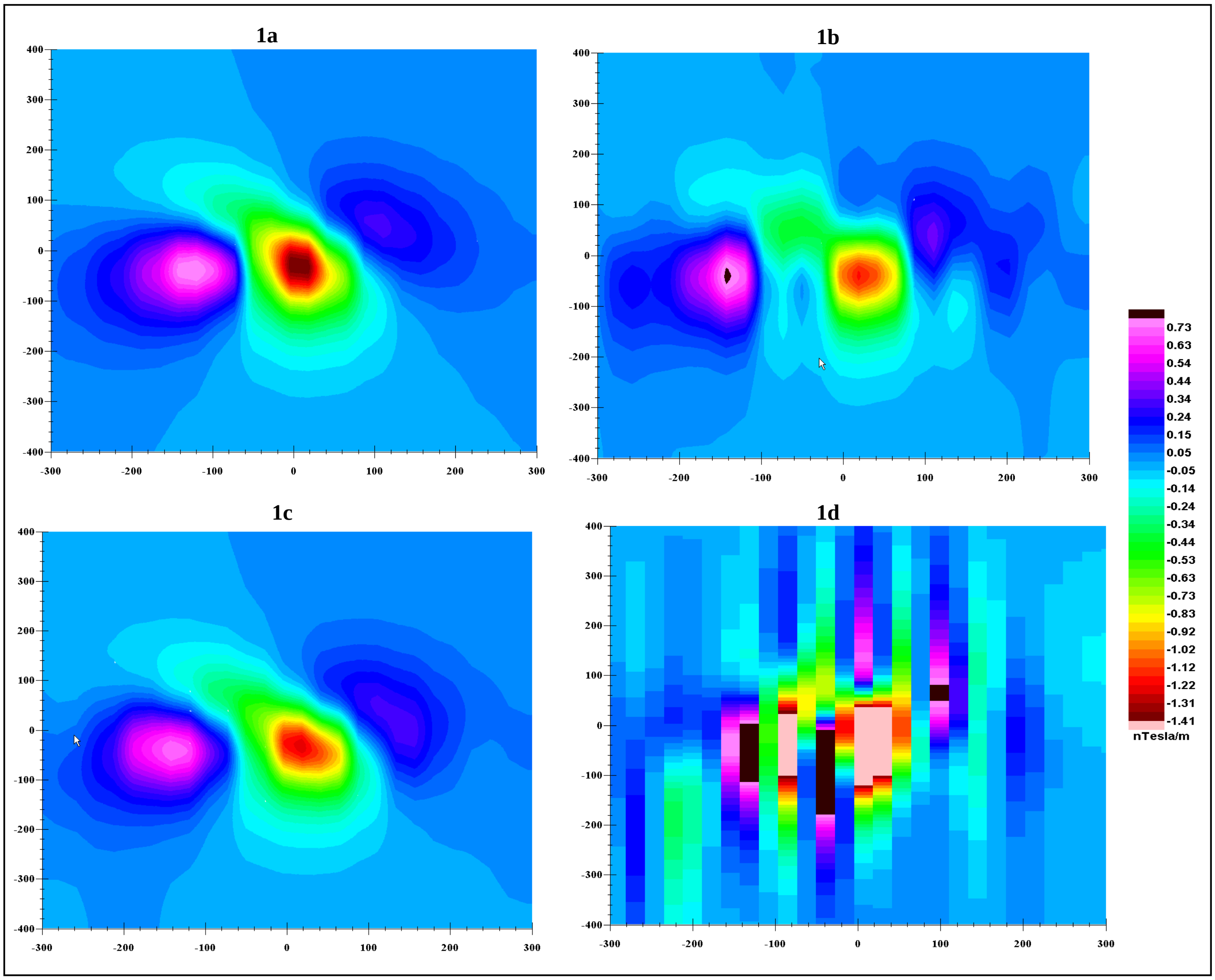


**Figure 1: Cross line derivatives without noise.**
**1a: true derivative, 1b: FFT derivative,**
**1c: ES at 80m, 1d: ES at 20m**

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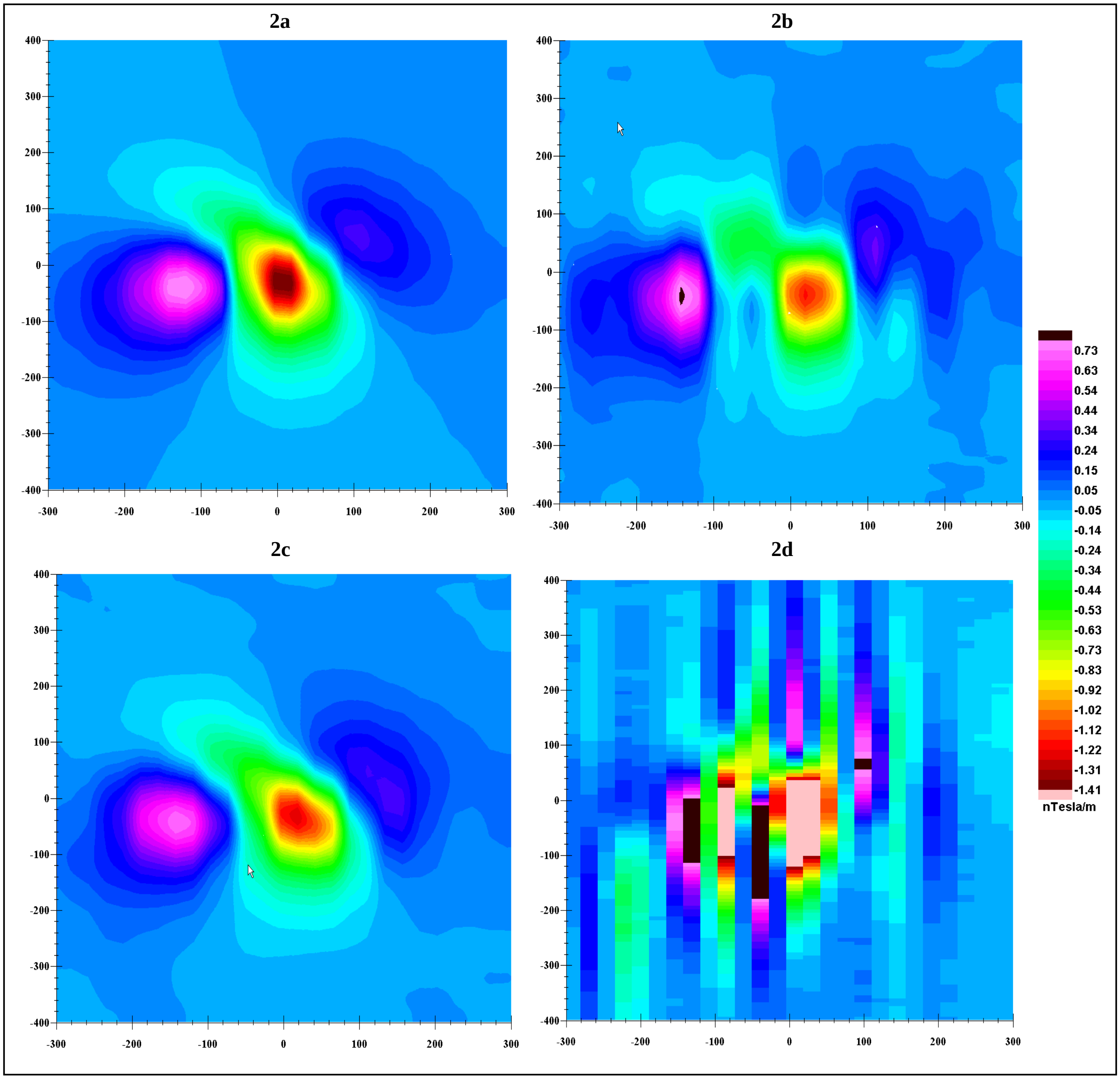


**Figure 2: Cross line derivatives with noise.**
**2a: true derivative, 2b: FFT derivative,**
**2c: ES at 80m, 2d: ES at 20m**