On Extending the Localized Nonlinear Approximator to Inductive Modes
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Abstract

Approximate methods have been widely employed to solve scattering problems in geophysics. They are particularly useful for simulating complex physical models in small geophysical environments where computing resources are limited. To devise an approximate theory, it is natural to start from the general integral equation for electromagnetic scattering

$$E(r) \ni E_b(r) \% \int_{V_s} \left(\bar{\bar{I}} \% \frac{\Lambda\Lambda}{k_b^2} \right) \frac{e^{ik_b|r' \& r|}}{4B|r' \& r|} Q(r') E(r') dV \, . \qquad (1)$$

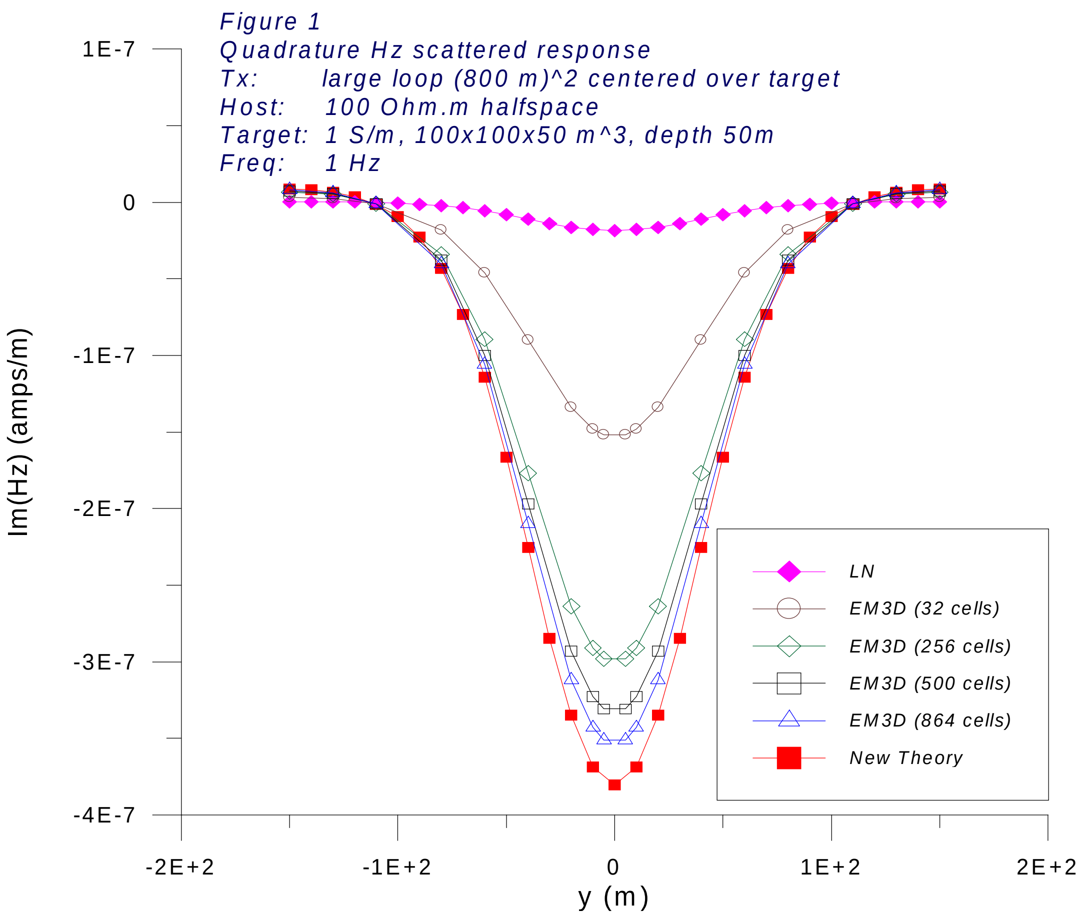
Assuming some functional form for the internal electric fields *E(r′)* in (1), one solves for the scattering currents and integrates them over the scatterers to obtain the response. The strength of the assumption on the form of the internal fields governs the usefulness of the resulting approximation. For example, the first order Born approximation [1] denotes the idealization of the scattering equation (1) in which the total field in the integral in (1) is approximated by the field in the background medium. The scattering currents are then obtained by scaling the internal field by the anomalous conductivity. In practice, the Born approximation is a weak scattering approximation that is appropriate if the difference between the internal electric field and the primary field are small (since it linearizes the problem in the conductivity contrast). On the other hand, the Localized Nonlinear *(LN)* approximation [2] uses the first (i.e., constant) term in a Taylor series expansion of the total internal electric field *E(r′)* about the target point *r* to approximate the field inside a scatterer, that is, $E(r') \approx E(r)$. One then simply solves a *3x3* complex linear system for the internal field and hence the scattering currents. Although it is nonlinear in the contrast, the *LN* approximation is accurate only when the scatterers are energized in a current channelling mode or when the excitation is weakly inductive. Recent theoretical developments have provided an extension of the *LN* algorithm to inductive modes while retaining the speed (i.e., *O(n)* complexity, where *n* enumerates internal sampling points) of the technique. The essence of the new theory is to

$$E(r') \approx E(r) + \frac{\partial E(r)}{\partial x}(x' - x) + \frac{\partial E(r)}{\partial y}(y' - y) + \frac{\partial E(r)}{\partial z}(z' - z) \, , \quad (3)$$

utilize an expansion to linear order of the internal electric field about the target point, i.e., whence one can formulate a *12x12* complex linear system for the unknown *E(r)* and its gradients at the target point *r* .

To illustrate the technique, we model a large loop inductive excitation (Figure 1): a block of conductivity *1 S/m* resides in a *100* Ω*.m* halfspace. An array of vertical magnetic dipole receivers at the surface bisects the target. Note the slow convergence of the University of Utah=s

EM3D solution, whereas the peak estimate of H_z from the present algorithm is altered by around *0.5%* in increasing the sampling from 75 to 108 points (only the former is shown). The simulation time for the new technique on a DX4 is about 6 minutes while the most densely sampled EM3D model on a Sparc 5 requires about 41 minutes. In addition, the EM3D makes use of four-fold symmetry, while the new theory does not. If symmetry is not used (for example, if the model was not symmetric) the EM3D computation time is further increased by a factor of 64.
Comparisons with other existing scattering algorithms including robust sphere and thin-sheet



techniques have shown the new method to be very effective at modelling inductive responses. Computation times for the technique are significantly faster than algorithms designed in the last decade specifically for speed. In addition, there is no breakdown of the solution in the near-field as is often the case for techniques of traditional design utilizing polynomial basis functions, since the new theory uses no explicit basis to represent the current density.

References

[1] M. Born, *Optik*, Springer-Verlag, New York (1933).
[2] T. M. Habashy, R. W. Groom and B. R. Spies, *Beyond the Born and Rytov Approximations: A Nonlinear Approach to Electromagnetic Scattering*, Journal of Geophysical Research, *98*, no. B2 (1993).