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SOME EFFECTS OF MULTIPLE LATERAL INHOMOGENEITIES IN MAGNETOTELLURICS[1]

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ABSTRACT

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The analytical solution, for the H-polarization magnetotelluric impedance, of a series of multiple, vertical, conducting slabs (dikes) embedded in a host medium is extended to an infinite array in order to model an anisotropic layer. The solution is used to study the effects of such strongly anisotropic media on the surface impedance. At low frequencies such vertically laminated structures behave as a bulk material. It is shown that the effective bulk parameters are those expected from d.c. theory. However, practical electrode separations may not be long enough or adequately positioned to correctly obtain these bulk parameters from the measured impedance. Thus, such structures can masquerade as quite different one-dimensional structures. A haphazard use of long electrode spacings will not necessarily produce correct results.

INTRODUCTION

Conductivities within the Earth often vary by orders of magnitude laterally as well as vertically. The electromagnetic response of these variations generally involves an interaction between inductive and galvanic effects with the most dramatic spatial variations occurring in the electric fields. These variations cause significant problems in the interpretation of magnetotelluric data, and it is therefore useful to have analytical solutions for simple models of such structures.

The two-dimensional (2D) H-polarization mode is a relatively simple but still useful model as it maintains the two physical effects, their interaction and the marked effects on the electric field. We describe the analytical solution, in the H-polarization mode, to the effects of multiple, vertical, conducting slabs (dikes) embedded in a host medium. The solution is then extended to an infinite periodic array to model an anisotropic layer. Extreme electrical anisotropy can be developed by these models.

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The study provides some insight into the influences of multiple, horizontal, laterally bounded, inhomogeneities on magnetotelluric (MT) sounding curves. The solution method can be utilized to study the effects of such complicated media on the surface impedance, the effective bulk parameters of these media and the information that can be recovered by magnetotellurics. This is a particularly useful model to study, as we can examine two important problems which are presently of interest in MT. The first problem is how such quasi-anisotropic structures may mislead conventional 1D interpretation. Such structures have already been invoked to explain magnetotelluric results (Schmucker, 8th Workshop on EM Induction in the Earth and the Moon, Neuchâtel, Switzerland, 1986). Secondly, with this model we can study an experimentally important problem; how to obtain adequate spatial sampling of the electric field. The experimentalist would like to know what sorts and sizes of effects can arise over such structure from too small an electrode separation or from misplaced electrodes.

For an infinite array of dikes, an analysis was done to determine effective bulk medium parameters for such structures. Both from this analysis and from the computer solutions one can distinguish the important parameters and their effect on the MT sounding curves. These parameters include the host resistivity, the resistivity of the inhomogeneities, the resistivity-thickness product of the inhomogeneities, the basement depth and conductivity, and the measuring position. As well as varying from the laterally homogeneous response, the surface response can vary dramatically from position to position.

It is then shown that the thickness of the anisotropic layer can be correctly determined by MT sounding curves if the layer is taken to have a uniform resistivity which is equal to its effective resistivity. With correct spatial sampling of the electric field, we can also obtain the correct effective resistivity by the MT method. However, it is also shown that, for such elongated structures, incorrect sampling of the electric field can produce apparent resistivities which are very different from the effective bulk resistivity. The results show the MT method to be an effective method for determining both the appropriate bulk conductive parameters and the correct thicknesses of significantly inhomogeneous media. This is especially true if the complex material is buried at some depth.

Lateral structures can masquerade as 1D structure. For example, the H-polarization response, obtained over a conducting host which contains resistive dikes, seems to indicate a conducting layer at depth when this data is interpreted only in 1D. A change in measuring position or long electrode spacings will not always help, because in some structures even large changes in measuring position will not detect the lateral structure.

The Effects of a Finite Number of Vertical Dikes

Rankin (1962) adapted the Fourier technique of d'Erceville and Kunetz (1962) for a single vertical contact to obtain the H-polarization solution for a vertical, conducting, rectangular prism of infinite length embedded in a host material. Wait and Spies

(1974) proposed extending the Rankin model to multiple dikes. However, although they provided some of the mathematical development for multiple structures with a perfectly insulating basement, they restricted their results to only one dike. (Wait and Spies (1974) provided normalized responses as functions of position across the single dike.) Here, the method of Rankin (1962) is first extended to obtain the solution for multiple, vertical structures or dikes. In particular, we initially examine the problem of a sequence of N dikes of horizontal thickness d and resistivity ρ_d embedded in a host medium of resistivity ρ_h. Each dike is separated by a distance h and the inhomogeneity structure has a depth D. The general model is illustrated in Fig. 1. The model here is restricted to only two basement resistivities (ρ_B), zero and infinity. Recall (Jones and Price 1970) that for the H-polarization mode

$$\mathbf{H} = H(x, z)e^{i\omega t}\hat{y}, \tag{1a}$$

$$\nabla^2 H = i\sigma\mu_0\omega H = \alpha^2 H, \tag{1b}$$

$$\sigma E_x = -\frac{\partial H}{\partial z}, \tag{1c}$$

$$\sigma E_z = \frac{\partial H}{\partial x}. \tag{1d}$$

Following Rankin, in any region labelled i (Fig. 1), let

$$H_i(x, z) = H_i^0(z) + P_i(x, z), \qquad i = 1, 2N + 1, \tag{2}$$

where $H_i^0(z)$ is the layered solution for region i if no lateral variation existed and $P_i(x, z)$ is the secondary magnetic field in the same region. Each perturbation function satisfies the 2D Helmholtz equation (1b)

$$\nabla^2 P_i = \alpha_i^2 P_i \tag{3}$$

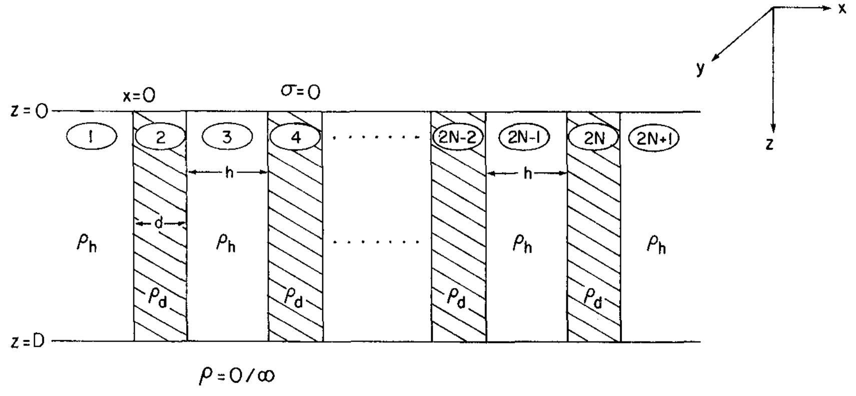

FIG. 1. An illustration of the 2D model for horizontal anisotropy which contains N dikes. The dikes have thickness d and are separated by a distance h. The resistivity of the dikes is ρ_d and that of the host, ρ_h. The inhomogeneity structure has a depth of D m. The upper resisting half-space represents the Earth's atmosphere.

in each homogeneous region. The differential equation (3) is separable and thus the perturbed solutions have the form

$$P_i(x, z) = \sum_{n=1}^{\infty} f_{in}(x) \sin \lambda_n z, \tag{4}$$

since the perturbed solutions are zero at the surface, $(z = 0)$. Here

$$\lambda_n = \frac{n\pi}{D} \rho_{\mathrm{B}} \text{ if } \rho_{\mathrm{B}} = \infty$$

and

$$\lambda_n = (n + \tfrac{1}{2}) \frac{\pi}{D} \rho_{\mathrm{B}} \text{ if } \rho_{\mathrm{B}} = 0.$$

From (4) and (3)

$$f_{in}(x) = a_{in} \mathrm{e}^{-\sqrt{\lambda_n{}^2 + \alpha^2} x} + b_{in} \mathrm{e}^{-\sqrt{\lambda_n{}^2 + \alpha^2} x}. \tag{5}$$

The question of why the spectrum contains only discrete (meaning the spectrum has the cardinality of the integers) eigenvalues (λ_n) arises. In other words, why is there not a continuous (cardinality of the real numbers) set of eigenvalues in the spectrum. That is, we would expect that in the most general case the solution would consist of a Fourier series as in (4) as well as a Fourier transform. In fact, it can be shown explicitly (Groom 1988) that for a horizontally bounded region of width L and arbitrary basement conductivity, there is an additional contribution, P_i^2, to the secondary magnetic field given by

$$P_i^2(z, x) = \sum_{n=1}^{\infty} \sin \frac{n\pi x}{L} \frac{\sinh \beta_n z}{\sinh \beta_n D} \frac{2}{L} \int_0^L P_i(D, s) \sin \frac{n\pi s}{L} \, \mathrm{d}s. \tag{6a}$$

Here, $P_i(D, x)$ is the secondary magnetic field along the interface between the bounded region and the basement. For an infinite region extending laterally from $x = 0$ to $+\infty$, this contribution becomes a Fourier transform

$$P_i^2(z, x) = \frac{2}{\pi} \int_0^{\infty} \sin (qx) \frac{\sinh \beta(q) z}{\sinh \beta(q) D} \hat{P}_i(D, q) \, \mathrm{d}q, \tag{6b}$$

with a similar solution if the region is extended to $-\infty$. $\hat{P}_i(z = D)$ is the Fourier transform of the secondary magnetic field along the interface between the semi-bounded region and the basement. First note that if the basement resistivity is infinite, then $P_i(D, x) = 0$ everywhere and P_i^2 is identically zero for both bounded (6a) and unbounded (6b) regions. Secondly, although this additional contribution is zero at the surface of the earth, the vertical derivative and thus the horizontal electric field at the surface is not necessarily zero. In particular, when the basement conductivity is infinite, the perturbed magnetic field is not zero everywhere along the basement contact and this additional contribution to the secondary magnetic field, which in turn contributes to the horizontal electric field, is not necessarily zero. Analytically, this contribution for the infinitely conducting basement has been ignored by previous authors. However, by comparison with numerical solutions for a single contact with a non-infinite but moderately high basement conductivity (Groom 1988), the contribution of the continuous spectrum (6b) was not found to be significant except near the basement. Here, we require solutions only at the surface ($z = 0$). Therefore for the perfectly conducting basement, the possible contribution of either (6a) for a finite region or (6b) for the semi-bounded region will be ignored.

If the basement is perfectly conducting then $f_{in} = 0$ when n is even to ensure that the horizontal electric field is zero at the interface $z = D$. Also

$$b_{1n} = 0, n = 1, 2, \ldots, \tag{7a}$$

$$a_{2N+1,n} = 0, n = 1, 2, \ldots, \tag{7b}$$

which ensures that the fields decay to zero correctly as the lateral position increases to infinity (i.e. $|x| \to \infty$). The remainder of the coefficients are obtained in a similar manner to the d'Erceville and Kunetz (1962) solution for a vertical contact solution. If there is a vertical contact at x_i, then equating the horizontal magnetic field for all depths, z at x_i results in

$$P_i(x_i, z) - P_{i+1}(x_i, z) = H^0_{i+1}(z) - H^0_i(z) = \Delta H^0_i(z) = \sum_{n=1}^{\infty} c_{in} \sin \lambda_n. \tag{8}$$

Then, using the orthogonality of the sine functions,

$$f_{in}(x_i) - f_{i+1,n}(x_i) = c_{in}, i = 1, 2N; n = 1, 2, \ldots. \tag{9}$$

The Fourier coefficients c_{in} are determined simply from (8) since they are the coefficients of a known function (ΔH^i_0). A similar relation between the f_{in} are obtained by equating the tangential component of the electric field (vertical field) at each conductivity-contrast contact. In this way, a system of $4N$ linear equations with $4N$ complex coefficients (a_{in}, b_{in}) is developed for each Fourier component. The $4N$ coefficients (a_{in}, b_{in}) describing the perturbed fields are then found by solving this complex matrix equation.

Figure 2 is an example of surface impedances (E_x/H) from such a model. This particular model consists of five relatively conducting dikes ($\rho_d = 10$ Ωm, $d = 500$ m) separated by a distance of 500 m in a host ($\rho_h = 1000$ Ωm). The depth of the dikes is 1000 m and the impedances were calculated at a frequency of 100 Hz. This example was chosen as it shows intermediate behaviour between the high frequency response which sees only the very local material and the low frequency response which is considerably affected by the basement conductivity. At this frequency, the skin-depth in the resistive material exceeds the width of the resistive structures whereas the skin-depth in the conducting material is less than the width of the conducting structures. The choice of basement resistivity is not relevant here since, at this frequency, the depth of the structure is much larger than skin-depth in either the host or the dikes.

The impedances in Fig. 2 are presented as a function of position along the top surface of the structure ($z = 0$). The entire structure begins at 0 and ends at 5000 m. The impedance magnitudes are expressed as apparent resistivities while the complex

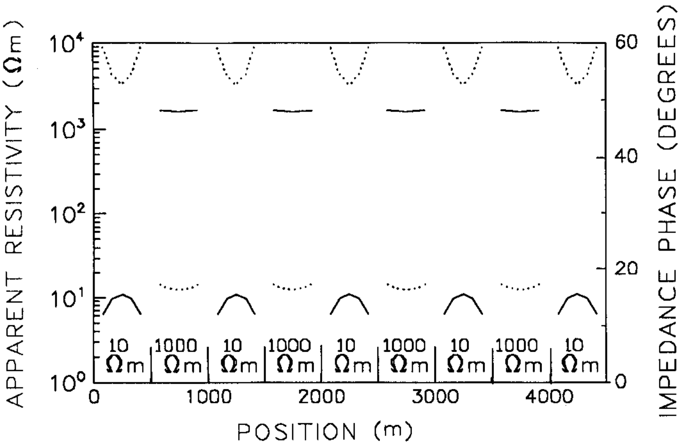

FIG. 2. Surface impedance across five vertical pairs in terms of apparent resistivity and phase at 100 Hz. The parameters are $h = d = 500$ m, $\rho_d = 10$ and $\rho_h = 100$ Ωm. The half-space is perfectly resisting and at a depth of 1000 m. (——) apparent resistivity, (·····) impedance phase.

phase of the impedance (MT phase) are given in degrees. A number of effects can be seen in this example (Fig. 2). Firstly, the apparent resistivity is suppressed over the relatively conductive dikes while it is slightly enhanced over the resistive host. Secondly, the impedances are sometimes a function of position (e.g. over the conductive dikes). Significant lateral gradients in the impedance occur only when the distance to the nearest vertical contact is less than one skin-depth in that medium. As frequency decreases to a point where the entire width of the slab is significantly less than a skin-depth, the impedance over a particular vertical structure (whether host or dike) becomes independent of position (e.g. as over the resistive host in Fig. 2). The suppression of the apparent resistivities over the conductive material was found to be a function both of the frequency and the 'resistivity × thickness' product of the resistive material.

THE EFFECTS OF AN INFINITE PERIODIC ARRAY

In the centre of the structure which contains five dikes, the impedances are essentially periodic with a period of 1000 m. This suggests a computationally more efficient but still informative model. Rather than a set of N dikes in a host, the model used was an infinite periodic sequence of vertical structures of alternating resistivity, ρ_d and ρ_h, and widths d and h. In this manner, a model for an anisotropic layer over a half-space is developed. The solution follows from the modified solution for multiple dikes. An advantage of the periodic model is that the fields are much quicker to solve numerically and thus the model provides a means for more rapid study. Since the structure is entirely periodic over all values of x then the fields on the left of these vertical doublets must equal the fields on the right. In this way only four complex coefficients are required for each Fourier component and thus only a 4 × 4 linear system must be solved to obtain each of the components. In addition, for high-order Fourier components there are approximations for the coefficients and the numerical solution can be obtained even more rapidly. This model also allows us to readily obtain the charge on the interfaces utilizing the continuity of normal current density and Gauss's Law. This can be useful since the suppression and enhancement of the electric field, which causes the variation in the apparent resistivities, is due mainly to gross polarization of the medium. Charge develops on the vertical interfaces and this charge produces a secondary field which either increases the electric field (in the resistive material) or decreases the electric field (in the conducting material).

Figure 3 shows how similar the solutions are for the two models. This figure utilizes the equivalent structural parameters for this periodic model, to those used for the finite dike model of Fig. 2. The modified structure now consists of alternating conducting ($\rho_d = 10\ \Omega$m) and resistive ($\rho_h = 1000\ \Omega$m) vertical dikes of 500 m width. The depth of vertically laminated material is that of the model for Fig. 2, namely 1000 m.

An Investigation of the Bulk Properties of the Model

The alternating periodic dike structure enables one to analyse the bulk properties of this extremely electrically anisotropic layer. Here the effective conductivity for horizontal current flow is quite different from that for vertical flow. As frequency

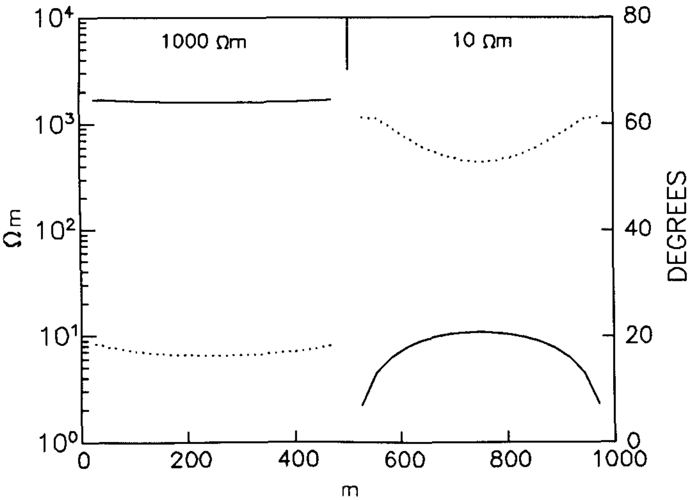

FIG. 3. The impedances across one vertical doublet of a periodic array which has a structure comparable to the finite number of dikes in Fig. 2. Each vertical structure is 500 m across. The resistive host is on the left while the conducting dike is on the right. (——) apparent resistivity, (· · · · · ·) impedance phase.

decreases, the horizontal electric fields at any given depth in such a composite layer will eventually become essentially constant as a function of horizontal position within each member of the pair. The entire field is then nearly horizontal and decays with depth. At such frequencies one can treat the anisotropic layer as a bulk medium. The frequency must be such that the skin-depth in both of the vertical structures becomes significantly larger than the widths in the respective material. This can be verified by evaluating the horizontal fields as a function of depth with this solution, and determining if the fields are constant across a dike at any given depth.

The effective d.c. resistivity of the medium is the spatially averaged resistivity

$$\rho_{\mathrm{eff}} = \frac{d\rho_{\mathrm{d}} + h\rho_{\mathrm{h}}}{d + h}. \tag{10}$$

It will be shown later that this is, in fact, the correct horizontal resistivity for such structure at low frequencies. (The concept that one vertical structure in the pair is the dike and the other the host is retained throughout this analysis.) Since current density is continuous across all these vertical interfaces, J_x will be constant as a function of x. We define an effective or average electric field by

$$J = \frac{1}{\rho_{\mathrm{eff}}} E_{\mathrm{ave}} \tag{11}$$

and the electric fields in each region can then be described in terms of this average or effective electric field. Thus, in the dike

$$E_{\mathrm{d}} = \rho_{\mathrm{d}} J = \frac{\rho_{\mathrm{d}}}{\rho_{\mathrm{eff}}} E_{\mathrm{ave}}, \tag{12}$$

while in the host

$$E_{\mathrm{h}} = \rho_{\mathrm{h}} J = \frac{\rho_{\mathrm{h}}}{\rho_{\mathrm{eff}}} E_{\mathrm{ave}}. \tag{13}$$

From voltage considerations

$$dE_{\mathrm{d}} + hE_{\mathrm{h}} = (d + h)E_{\mathrm{ave}}, \tag{14}$$

and thus it is easily shown that (12–14)

$$E_{\mathrm{ave}} = \frac{1}{d + h}\left[d + h\frac{\rho_{\mathrm{h}}}{\rho_{\mathrm{d}}}\right]E_{\mathrm{d}} = \beta(d)E_{\mathrm{d}} \tag{15a}$$

$$= \frac{1}{d + h}\left[h + d\frac{\rho_{\mathrm{d}}}{\rho_{\mathrm{h}}}\right]E_{\mathrm{h}} = \beta(h)E_{\mathrm{h}} \tag{15b}$$

Therefore, in the resistive limit the apparent resistivity over the dike is given by (15a)

$$\rho_{\mathrm{app}}(d) = \frac{1}{\mu_0\,\omega}\left|\frac{E_{\mathrm{d}}}{H}\right|^2 = \frac{1}{\beta^2(d)}\frac{1}{\mu_0\,\omega}\left|\frac{E_{\mathrm{ave}}}{H}\right|^2 = \frac{1}{\beta^2(d)}\rho_{\mathrm{eff}} \tag{16}$$

and similarly over the host (15b)

$$\rho_{\mathrm{app}}(h) = \frac{1}{\beta^2(h)}\,\rho_{\mathrm{eff}}\,. \tag{17}$$

If the dikes are resistive relative to the host then $\rho_{\mathrm{h}}/\rho_{\mathrm{d}} < 1$. Therefore, $\beta(d) < 1$ and $\beta(h) > 1$; thus the obvious conclusion is reached that

$$\rho_{\mathrm{app}}(d) > \rho_{\mathrm{eff}}\,, \tag{18a}$$

$$\rho_{\mathrm{app}}(h) < \rho_{\mathrm{eff}}\,. \tag{18b}$$

The variations from the effective resistivity are not only dependent on the ratio of resistivities but the ratio of the widths. For example, a thin resistive dike produces a relatively large increase in apparent resistivity over the dike. On the other hand, a thick resistive dike produces a large decrease in apparent resistivity over the host. Clearly, if the dikes are conducting, these relations are reversed.

The Frequency Response of the Model

The results of the above analysis indicate a low-frequency behaviour for the medium which may not be surprising to the reader. The high frequency result is also obvious as the apparent resistivity will approach the resistivity of the local material. However, this model and solution enables another aspect of magnetotelluric behaviour to be studied; namely the response in the transitional frequency range between the low and high frequency behaviour. In particular, to study whether the impedance response can be modelled in 1D and if so whether false conducting or resistive layers could thus be inferred. One could argue that such incorrect inferences would never be made, as 1D models should not be used to fit impedance tensors which are observed to be 2D. However, small-scale 3D inhomogeneities will often cause a ‘mixing’ of the E- and H-polarization impedances of a 2D structure (Groom 1988). As a possible consequence (Groom 1988) a genuine 2D structure can appear as a 1D structure with ‘static shift’. The interpreter might then be led to interpret the data with a 1D model.

Figure 4 is an example computed using the method described in this paper. It is used to illustrate the effects of such horizontally laminated structure. A plot (Fig. 4) is given of apparent resistivity versus frequency at points over both a resistive region and a conducting region. In this example, which models a set of resistive dikes in a more conducting host, $d = 50$ m, $\rho_{\mathrm{d}} = 1000\ \Omega$m, $h = 550$ m and $\rho_{\mathrm{h}} = 10\ \Omega$m. The depth of the structure was chosen to be 15 km to clearly illustrate the effects.

Firstly, note (Fig. 4) that over the resistive dike at all frequencies sufficiently high not to be significantly affected by the basement, the apparent resistivity is enhanced above the resistivity of the dike. Over the relatively conducting host, the apparent resistivity is suppressed as expected from (18). (The measurement point is exactly in the middle of each member of the doublet structure.) One can see the frequency-dependent effect of the structure. At high frequencies, each member of the structure is seen individually. As frequency decreases, a transition to impedances due to the

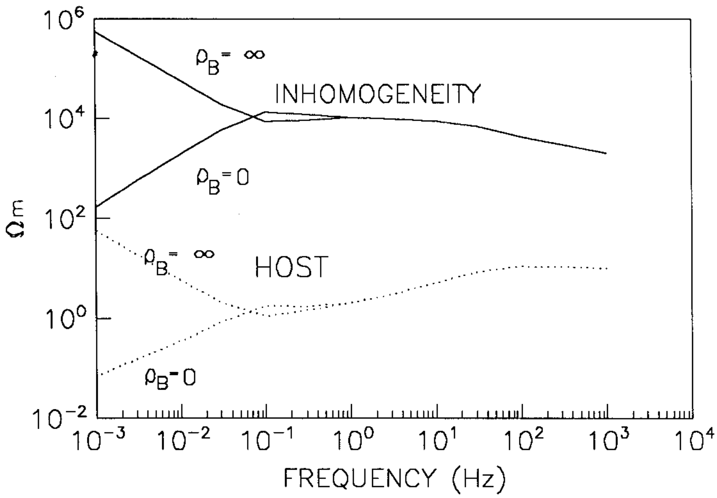

FIG. 4. Surface apparent resistivity as a function of frequency at two sites. One site at the centre of resistive inhomogeneity (d = 50 m, ρ_d = 1000 Ωm) while the other is at the centre of conducting host (h = 550 m, ρ_h = 10 Ωm). The depth of the anisotropic layer is 15 km.

bulk properties is made, and finally at low frequencies the basement resistivity dominates. The results are shown for both zero and infinitely resistive basements. In the low frequencies the apparent resistivities sweep upwards for the infinitely resistive basement and sweep downwards for the infinitely conducting basement.

It is important to note that, as frequency decreases, the laminated structure is thick enough that the apparent resistivities reach the bulk effective resistivities *prior* to significant effects due to the basement resistivity. This is important as it means that the truncation of the anisotropic layer by the lower half-space has not interfered with the nature of the transition of the anisotropic layer's response from its high frequency response to that of a bulk medium. Equations (15)–(17) predict the apparent resistivities when the anisotropic layer is acting as a bulk medium and the basement has negligible effect. For example, at 0.1 Hz where the laminae thicknesses are much less than a skin-depth in the respective media but the depth of the laminated layer considerably exceeds a skin-depth, the figure agrees well with the predicted bulk properties. Over the conducting region, these equations predict that by 0.1 Hz the apparent resistivity should be 1.08 Ωm while over the resistive region it should be 1.8×10^4 Ωm. The bulk effective resistivity (10) for this model is 92.5 Ωm. Figure 4 shows that the apparent resistivities level off very close to these values before being swung upwards or downwards at still lower frequencies by the basement resistivity.

Now consider how the basement is revealed by the impedance response in Fig. 4. The frequency at which the apparent resistivities begin to change rapidly to approach the basement resistivity is 0.1 Hz. At this frequency the fields just penetrate to the basement and thus contain information about the apparent depth to basement. If the material were homogeneous this would happen at a frequency where the thickness of the layer was approximately one skin-depth. At 0.1 Hz, the skin-depth in a material having a resistivity equal to that of the effective resistivity (92.5 Ωm) would be approximately 15 km. Whereas 15 km is approximately one skin-depth at 1 Hz in the resistive dikes and one skin-depth at 0.01 Hz in the conducting host. Thus, we can conclude that at 0.1 Hz the anisotropic layer is acting like a bulk material having a resistivity equal to that of the effective resistivity. In particular, the correct thickness can be determined if the effective bulk resistivity is utilized. However, the shift of the apparent resistivities up or down from the effective resistivity is strongly a function of position; considerable averaging over measuring sites or possibly very long telluric lines would be required to evaluate it. That the apparent resistivities over the dike and host can be predicted from bulk considerations also indicates that the layer is acting as a bulk material. How to evaluate the effective bulk resistivity and the correct thickness of the anisotropic layer will be considered below.

Finally, note that over the conducting host, the thin resistive dikes have the effect of producing what appears to be a conducting layer at depth when the curves are interpreted only in 1D as shown in Fig. 5. As an example of how well this type of data can be modelled in 1D, the sounding data over the conducting region from the previous model (Fig. 4) are inverted and the fit to data shown in Fig. 5. The inversion of data for the resistive basement was used in this plot. The 1D model chosen was two layers over a half-space with the resistivity of the top layer constrained to be 10 Ωm. Arbitrarily, a phase error standard deviation of 2° and an apparent resistivity standard deviation of 10% were added to simulate real data

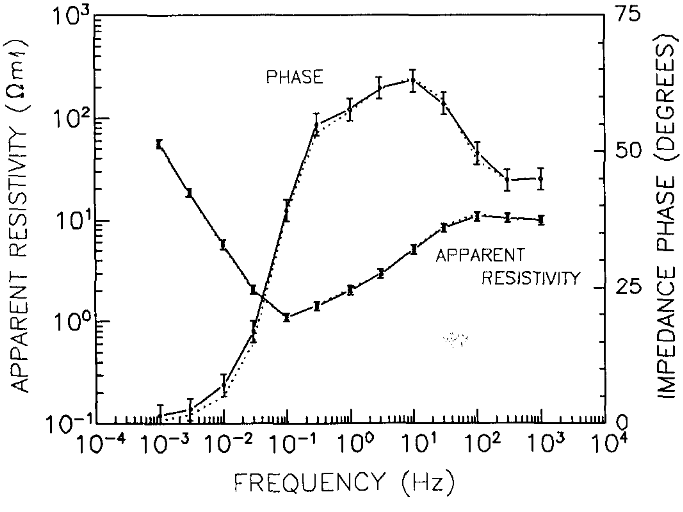

FIG. 5. *1D inversion of 2D data. The data is that of the previous model (Fig. 4) from the site* over the conducting host. The inversion model contains two layers over a half-space. The 1D model parameters are $d_1 = 192$ m, $d_2 = 1707$ m, $\rho_1 = 10$ Ωm, $\rho_2 = 1.14$ Ωm, $\rho_B = 10^6$ Ωm. The error bars give one standard deviation in the synthetic data. (——) 2D data, (······) 1D fit.

with noise. The model parameters which fitted the impedance curves of Fig. 5 are $d_1 = 192$ m, $d_2 = 1707$ m, $\rho_1 = 10\ \Omega$m, $\rho_2 = 1.14\ \Omega$m and $\rho_B = 10^6\ \Omega$m.

Although only the resistive basement data is used for Fig. 5, the inversions were done with both basement resistivities to compare the results. Except for the basement resistivity in the 1D model, the 1D parameters for the two basement conductivities are very similar, which indicates that the basement conductivity was not a factor in the parameters for the false conducting layer. For the conducting basement at the same site, the best fitting model parameters are $d_1 = 188$ m, $d_2 = 1919$ m, $\rho_1 = 10\ \Omega$m, $\rho_2 = 1.2\ \Omega$m and $\rho_B = 10^{-6}\ \Omega$m. Thus, for this particular model, the apparent resistivities and phases at the host measuring position produce 1D models which have a false conducting layer at depth.

Interpretation of the other polarization (E-polarization) for such a model would prevent such misinterpretations by indicating an electrical anisotropy at depth. The question that then arises is whether the response is due to 2D or 3D structure, or 1D anisotropy. The 1D anisotropic model may be important in practice. Schmucker (1986 EM Induction Workshop, as above) observed such quasi-anisotropy in the crust or upper mantle under Western Germany and attributes it to a series of highly conducting dikes. R. Kurtz (private communication) also has data which may indicate deep anisotropy in the mantle under Ontario.

Experimental Evaluation of the Bulk Parameters

How should one actually determine the correct bulk parameters from impedance measurements? If one does not correctly measure the effective bulk resistivity then the thickness will be determined incorrectly. The ideal solution, of course, is to correctly measure the *average* electric field (11), but in the absence of *a priori* knowledge of the inhomogeneities this can be difficult experimentally.

The presented laminated model and solution allow this problem to be studied. Two scales for the inhomogeneities will be considered: in one the widths of the dikes are significantly less than the electrode spacing, while in the other the dike widths are comparable with the electrode spacing.

Figure 6 illustrates how electrode placement, for measuring the electric field, determines the apparent resistivity obtained. In this model, $d = h = 1$ km, $\rho_d = 10$ and $\rho_h = 1000\ \Omega$m. The depth of the structure is 5 km and the basement resistivity is zero. Equation (10) tells us that the effective resistivity is 505 Ωm while (16) indicates the apparent resistivity of the conducting dikes is 0.2 Ωm while over the resistive host it is 1980 Ωm. Figure 6 shows the apparent resistivities and phases for five different electrode placements. All the electrode lengths are 1000 m. The placements range from the electrode being entirely over the conducting region (0×1000), through three-quarters over it (250×750), half (500×500), one-quarter (750×250), to entirely over the resistive region (1000×0). It can be seen from Fig. 6 that one sampling (500×500) of the electric fields can give the correct bulk resistivity (505 Ωm). This occurs here when one samples equally over both structures.

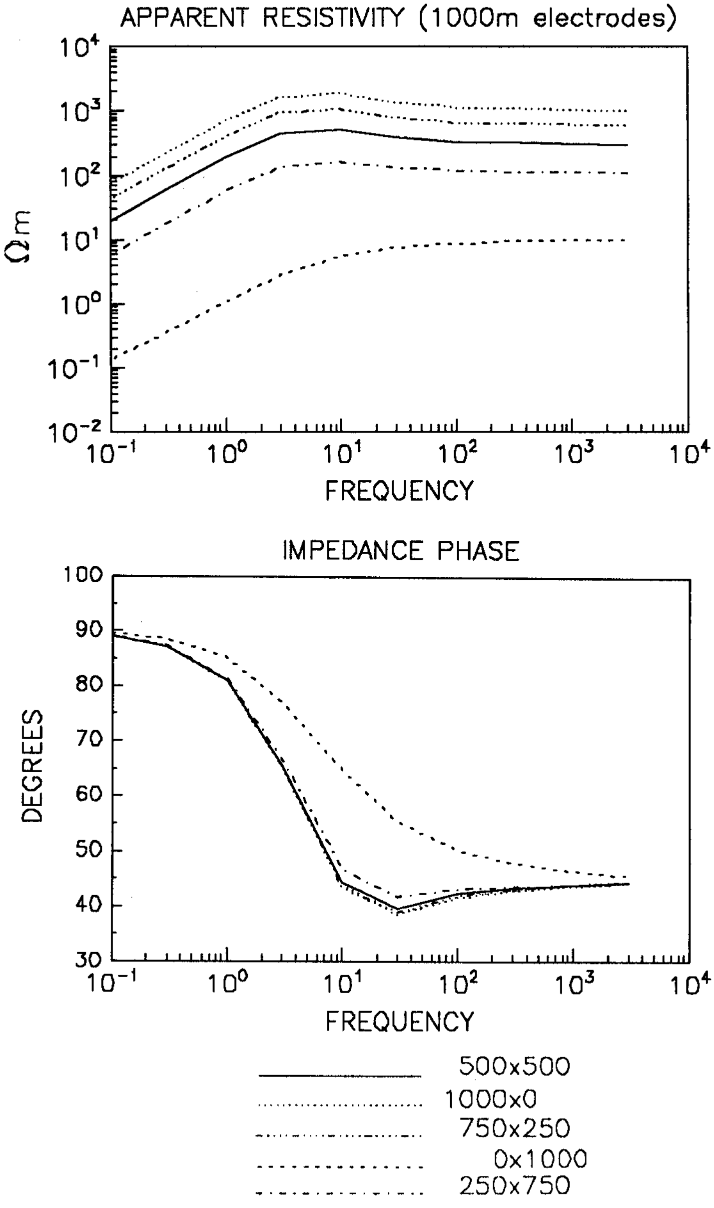

FIG. 6. A study in the use and abuse of long electrodes to measure the electric field. In this model $d = h = 1$ km, while $\rho_h = 1000$ Ωm, $\rho_d = 10$ Ωm, $D = 5$ km and the effective bulk resistivity (ρ_{eff}) is 505 Ωm. The electrode length is 1 km and the surface impedances are shown for five different electrode placements.

However, misleading bulk parameters are obtained if the electrodes are incorrectly placed. The errors occur mostly in the apparent resistivities, except when there is no sampling of the electric field over the resistive part of the structure. Then significant phase deviations occur. The resulting apparent resistivities are shifted from those of the bulk response by more or less frequency-independent factors. The phase responses, on the other hand, are virtually identical except or where the electric field was sampled simply over the conducting material. Finally, note that at 5 Hz one skin-depth in the bulk material (505 Ωm) is 5058 m. This is the frequency at which the model begins to turn significantly downwards, towards the basement resistivity (zero). The conclusions from this model are that, although long electrode spacing can help, for elongated structures the method must be used with care.

Finally, a different scale of inhomogeneity is studied where the electrode separation is much greater than lamina thickness. This models inhomogeneities which are small compared to the electrode length. The resistivity contrast is again the same as in the previous models (10–1000 Ωm). The thickness of the dikes is 10 m and they are separated from each other by 10 m of host. The depth of the anisotropic layer is 2000 m. Again the effective resistivity is 505 Ωm (10). With averaging over several laminae or doublet pairs we expect reasonable estimates of bulk parameters. Figure 7 presents the impedances when the electrode length is 110 m. With a fixed electrode spacing, the impedances are only slightly dependent on position. For this reason, the response for only one particular placement of the electrodes is plotted. The electrode covered five doublet pairs and then extended 10 m onto the conducting host. For an electrode of this length, this layout is expected to have the largest variation from the effective resistivity. Figure 7 shows, however, that this electrode layout determines a value very close to the correct bulk resistivity. The skin-depth in 505 Ωm material is 2064 m at 30 Hz. This is the frequency at which the basement begins to significantly affect the apparent resistivity. Thus, in this case, the spatial sampling of the electric field has been adequate to produce both an apparent

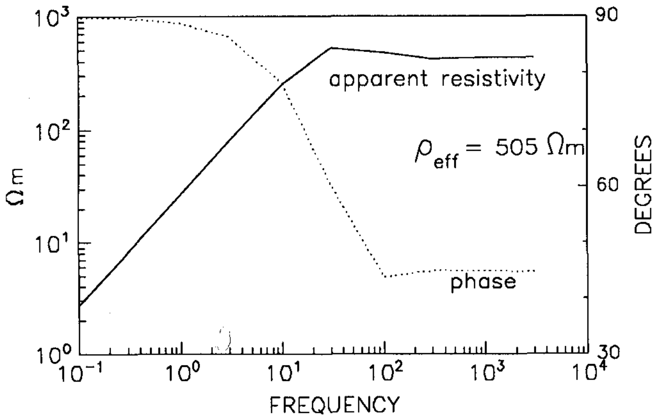

FIG. 7. Fine structure impedance with 110 m electrode. In this model $d = h = 10$ m while again $\rho_h = 1000$ Ωm and $\rho_d = 10$ Ωm. The electrode covers five doublet pairs and one more conducting structure.

resistivity which is appropriate for the material (the effective resistivity) and the correct thickness of the layer when the correct bulk parameter is used.

Summary

A solution has been provided for the MT H-polarization impedance over a strong horizontally anisotropic structure which models the frequency dependence of the transition from the influence of the individual media to that of the bulk medium.

For structures of this type it is shown that at sufficiently low frequencies, the material behaves as a uniform material with effective bulk properties. The correct bulk resistivity is its effective resistivity (10). The model has indicated that for elongated structures MT can obtain useful parameters if the electric field is sampled correctly. However, incorrect sampling can produce erroneous results. These erroneous results are dependent upon frequency, the host resistivity, the depth of the inhomogeneous layer and the 'resistivity × thickness' product of the inhomogeneities. A haphazard use of long electrodes will not necessarily produce useful results. It is also shown that in the presence of resistive dikes, impedance curves obtained over the more conducting host can produce false conducting layers if the curves are interpreted only in 1D. Experimentally, these false interpretations can be difficult to detect because of the mixing of polarizations by 3D inhomogeneities.

A conducting layer overlying such a structure will have the same effects on the measured impedance as the use of a long electrode. That is, the electric fields over the two laminae will become more uniform by the averaging effect of the diffusing through the upper layer. The amount of averaging will depend upon the relative thickness of the layer and the width of the doublet. If the overlying structure is sufficiently thick, the electric fields measured on the Earth's surface should be the averaged electric field. Thus, when the anisotropic layer is buried at sufficient depth, magnetotellurics perceives the structure in a 'reasonable' manner. However, the results indicate that if resistive dikes do not reach the surface a false conducting layer could be indicated on the sounding curves without any surface signature for the dikes. This will occur when the overlying layer is not electromagnetically thick and the distances between the contacts are relatively large compared to the electrode spacing.

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