Electromagnetics 2: MT and EMAP

Wednesday Morning

Quantitative Methodology for Determining the Dimensionality of Structure

EM 2.1

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SUMMARY

A methodology is described to evaluate quantitatively the dimensionality of the conducting structure which governs a set of MT data. This methodology is based upon three general models or parametrizations of the MT tensor, each of which has a different physical interpretation.

The methodology utilizes a weighted statistical residual which describes the fit of the model response to the data within the scatter of the measured data. The method has been tested successfully with synthetic data. With field data, the methodology is not always straight-forward but can answer many questions. This is illustrated by an example with field data.

INTRODUCTION

With the increased use and sophistication of forward and inverse modelling schemes for conventional MT data, the ability to determine quantitatively the dimensionality of the conductivity structure is becoming increasingly more important. For example, one must determine if the structure is significantly 3D and to what extent the three-dimensionality affects 2D modelling. Also, due to the significance of near-surface effects, workers sometimes rotate their data to an assumed geo-electric strike and then invert either one or both of resulting off-diagonal elements of the MT tensor. It is critical to determine to what extent the data actually fits an assumed strike and two-dimensionality.

Determining this dimensionality is not trivial since the dimensionality can change with period. Large-scale features which appear 2D at short periods often become increasingly more 3D at longer periods. It is necessary to recognize these changes in dimensionality. With changing period not only the dimensionality of the earth but also the scale of the recording changes. For example, at long periods and skin-depths, even extended electric field measurements with electrode lengths of a few kilometres can become essentially point measurements when significant 3D structures have scale lengths of tens of kilometres or more. Regional strike directions change when a structure which was 2D at shorter periods becomes a 3D "static" distorting structure at longer periods. We must determine as much as possible about the dimensionality of the structure before conductivity modelling begins.

PHYSICAL MODELS FOR IMPEDANCE TENSOR

The simplest model of the earth is a 1D, layered model. In which case, the impedance tensor is estimated by the expression:

$$\hat{\mathbf{Z}}(\omega) = \begin{pmatrix} 0 & Z_0(\omega) \\ -Z_0(\omega) & 0 \end{pmatrix}. \qquad (1)$$

There are only 2 model parameters per frequency, namely the magnitude and complex phase of Z_0. We might wish to impose, as well, smoothing or causality constraints on the model parameters.

A more complicated model is to assume the conductivity structure is 2D. Thus, except for the presence of noise, the MT data tensor is estimated by:

$$\hat{\mathbf{Z}}(\omega) = \mathbf{R}(\theta) \begin{pmatrix} 0 & Z_{\parallel}(\omega) \\ -Z_{\perp}(\omega) & 0 \end{pmatrix} \mathbf{R}^t(\theta). \qquad (2)$$

In this case, we have 5 parameters per frequency (Swift, 1967) although it is possible to impose constraints on the regional strike (θ) by constraining it to a geological trend or to be independent of frequency.

It is also possible to have a 3D model to represent the principal effects of 3D conductive structure. If the frequency is low enough that the 3D structure has a negligible inductive response then the EM fields can be described, to first order, by regional (1D or 2D) electric fields being galvanically distorted and the regional magnetic field being unchanged. Thus the impedance tensor is described by

$$\hat{\mathbf{Z}}(\omega) = \mathbf{R}(\theta)\ \mathbf{C}(\theta) \begin{pmatrix} 0 & Z_{\parallel}(\omega) \\ -Z_{\perp}(\omega) & 0 \end{pmatrix} \mathbf{R}^t(\theta). \qquad (3)$$

where $\mathbf{C}(\theta)$ is the 3D galvanic electric scattering or distortion operator represented in the regional 2D co-ordinate frame (Bahr, 1988; Zhang *et al*, 1987; Groom and Bailey, 1989b). It is possible, via the techniques of Groom and Bailey (1989a,b), to decompose the data under this model to obtain 7 parameters per frequency; namely the regional 2D strike (θ), two parameters partially describing the effects of the local electric field distortion (twist and shear) and the 2D complex regional impedances ($Z_{\parallel}, Z_{\perp}$) (each possibly shifted independent of frequency by a real number). We may again wish to constrain the distortion parameters as well as the regional strike to be independent of frequency over some subsets of the frequencies.

These models (1)-(3) do not include all effects of all possible conductivity structures at all measured periods. We are not yet able to parametrize under a physical hypothesis the effects of 3D induction. Although the effects of 3D galvanic magnetic scattering can be parametrized, a method for extracting these parameters from the data has not yet been devised. We are however able to estimate both the magnitudes and decay of these effects on the data with increasing period and distance (*e.g.* Groom and Bailey, 1989a; Groom, 1988). In fact, except for very large structures, both the inductive and galvanic magnetic effects are usually secondary to those included in (1)-(3).

QUANTITATIVE TESTING FOR DIMENSIONALITY AND STRUCTURAL PARAMETERS

Having modeled the data to one of the three physical hypotheses (1)-(3) and obtained the model parameters (possibly constrained), we wish to quantitatively test the model hypothesis by a a χ^2 variable. This is a residual error of the fit of the model to the data, normalized by estimates of the variance (σ_{ij}^2) of each element of the tensor data:

$$\gamma^2 = \frac{1}{4}\sum_{i=1}^{2}\sum_{i=1}^{2}\frac{|\hat{Z_{ij}} - Z_{ij}|^2}{\sigma_{ij}^2} \qquad (4)$$

where $\hat{Z_{ij}}$ and Z_{ij} are the elements of the modelled and measured tensor, respectively. The variances would usually be estimated from the sample population of tensor estimates.

If the model parameters (1)-(3) fit the data within the noise, they should then fit almost always within 3 standard deviations of the data. γ^2 would therefore be expected to lie within the range $0 - 9$. If none of the 3 models fit within these levels, this could possibly imply that not all physical effects have been included (*i.e.* 3D induction). The χ^2 residual (γ^2) emphasizes the fit of the model elements to the corresponding data elements which have the smallest variance. However, if the variances are poorly estimated then this statistic (4) may be either biased up or down. In this case, we might seek the model with the lowest residual statistic. It must be emphasized that the testing must be done selectively over multiple frequencies as the statistic can vary significantly between adjacent periods due to either the randomness of the noise or poor variance estimates.

The best model is not necessarily the one with the lowest average residual (averaged over periods) but rather we must consider a trade-off between residual and smoothness. By smoothness, we mean the fewest number of parameters used over multiple frequencies. For example, under model (3) if the strike and the distortion parameters (twist and shear) are all constrained to be a constant over N frequencies, then we will have used $4N + 3$ independent parameters for N frequencies. On the other hand, if model (2) is used and strike is not constrained to be independent of frequency, we will have used $5N$ parameters and if $N > 3$ we have a rougher model. We could, of course, include additional restraints to smooth the model such as impedances which vary smoothly with period.

AN EXAMPLE WITH SYNTHETIC DATA

Synthetic data are generated by a combination of numerical and analytic techniques to create very accurate 3D MT data (Groom and Bailey, 1989a). A conducting slab (300 Ωm) vertically contacting a resistive slab (40,000 Ωm), both underlain at 10 km by a rather conductive half-space (10Ωm), is the basic 2D conductivity model. Numerical solutions are found for the TM and TE impedances. Into the 300Ωm slab is embedded a small conducting hemisphere, at 6 km from the vertical contact. The hemisphere is sufficiently small for the frequencies used that it has a negligible inductive response. The galvanic effects of the hemisphere on the fields produced by the large-scale 2D structure are found analytically via scattering operators. The numerical and analytic solutions are used together to synthesize 3D MT data at any site desired on the surface of model. Noise is added to the synthetic data. It is assumed the variance of the noise is identical for all 4 elements of the MT tensor. For the purpose of the discussion here, the measurement axes were taken to be parallel and perpendicular to the 2D strike.

A test site is chosen which is 16 m outside the hemisphere and at an angle of 22.5° counterclockwise to that measurement axis which is perpendicular to the 2D strike. We can, for example, examine the parameters and the residual for the 2D model (2). Figure 1 contains the strike direction recovered by the conventional least squares means and the residual (4). For periods longer than 0.1s, the data are fairly sensitive to the 2D contact. At this period, the strike direction changes from about 23° to 14° while at the same time the residual jumps from below 10 to 1000. Clearly, at least for the long periods, we would conclude from Figure 1 that the structure is significantly not two-dimensional.

Figure 2 presents some of the parameters under the 3D model (3). The noise is very small having a variance of only .01% of the magnitude of the largest element. (The figures present the χ^2 residual, the twist and shear and a normalized error as functions of possible 2D regional strikes and period.) The 3D parameters are obtained by constraining the strike at increments from $0 - 90°$ and applying the Groom and Bailey (1989b) 3D decomposition. A white shaded χ^2 error indicates a good fit. Although, the correct strike (either 0 or 90°) is determined in the middle periods, both the shortest and longest periods appear to show no preference for this parameter. At the shortest periods, this is explained by the fact that the the data is not sensitive to the 2D contact while at the longest periods the 2D structure is thin and has essentially no inductive response (*i.e.* the 2D TE and TM impedances have the same phase). At these long periods, the 2D impedance tensor can be approximately described by a 2D real distortion matrix times a 1D impedance tensor. Without information from shorter periods, the 2D distortion matrix in combination with the 3D distortion matrix (3) represents a net effect which cannot be distinguished from 3D distortion of 1D data. Constraining both the twist and shear to be independent of frequency gives a low χ^2 residual only for the correct 2D strike. The final sub-figure (relative error) in Figure 2 is the χ^2 residual (4) normalized to emphasize the best fitting (black) and the worst fitting strike (white). Note that the worst strike follows the 2D model estimate of strike (Fig 1).

The stability of parameters in the presence of noise is an important facet of this study. Figure 3 is an example of the same data as Figure 2 with the noise is increased to 2%. Notice that both the χ^2 residual and the normalized error are much less able to resolve the correct strike. However, the twist and shear angles have varied only slightly from their values in Figure 2. Again constraining twist and shear to be independent of frequency will result in a low χ^2 residual over all periods only for regional strike directions which are approximately correct.

In this synthetic example, the physics implies a relatively smooth model as the twist, shear and regional strike should be independent of frequency. This indicates the general result we seek, a trade-off between the smoothest model (*i.e.* least parameters) and the lowest average residual.

AN EXAMPLE WITH FIELD DATA

With actual field data this process can be more difficult because of poor data and possibly poor estimates of variance. Also, the structure may not be as simple as in the synthetic example or 3D induction may be important and we do not have the necessary physical parametrization. To indicate both the usefulness and the difficulties still remaining in this methodology, data from one of a number of sites obtained in the Canadian Southern Cordillera (Jones *et al.*, 1988) is used here. The general trend of the geology throughout the region is N-S and thus the data were collected with the measurement axes parallel and perpendicular to this direction. At the same time, the site (EMR000) lies in a valley which follows the Slocan Lake Fault and locally strikes about 30° to the NE. The valley contains shallow relatively conducting sediments.

Figure 4 contains the relevant parameters for the conventional interpretation (2) when the strike is chosen to be N-S. The χ^2 residual is uniformly large for all periods. There are two indicators of strong 3D effects. For periods longer than 1 s, the skew angle (arctan of skew) rises rapidly and the phase of the one of the estimated impedances increases from about 60 to 180°. The noise to signal ratio plot shows, for each element of the tensor, the magnitude of the square root of the estimate of the variance to the magnitude of estimated element. The magnitude of this ratio changes with different period ranges and implies the residual plots should be used in conjunction with this variance to mean ratio. Improved fits may have to be determined by comparing residuals for other models rather than by examining the residuals absolutely.

Figure 5 gives the residual error for the 3D model as a function of period and possible regional strike. This residual error indicates that at short periods a 2D strike of about 25° is preferred. This changes to about 0° in the midband and the strike direction seems indeterminate in the long periods. In fact, for the short periods an approximate 2D model with very small static 3D effects is the best model. This is reasonable as the valley and sediments should dictate strike in the short periods. In the mid-range, a strike approximately N-S with fairly strong but constant channelling parameters (twist and shear) fits the criteria of low residual and fewest parameters. The local strike (an estimate of the local current direction, Groom and Bailey, 1989b) indicates the current remains constrained to flow at about $25 - 30°$. Thus, the valley seems to constrain the current while the regional strike is more N-S.

The longest periods are more difficult to explain. With a N-S strike even with 3D distortion parameters results in one phase becoming exceptionally large at these periods. Is the underlying structure 1D with very strong local 3D effects? The residual (Fig 5) indicates this possibility. However, this can occur due to the 2D structure being inductively thin (Fig 2). Searching through possible long period parametrizations, it was found that a 2D strike of 25° leads to a consistent model (Figure 6). That is, the residual decreases rapidly below 10 seconds, the twist and shear are constant, both phases are within the expected quadrant and the local strike is approximately in the direction of the valley which seems to govern current flow at the site. The minor phase is very sensitive to small variations in strike direction.

It seems evident that small static effects at short periods become larger as the valley becomes a near-surface distorting body. The N-S strike which is probably due to the general trend of the resistive batholiths seems to disappear at longer periods while the distortion parameters increase. It is possible that the nearby batholiths have become 3D structures at long periods. The MT tensors have in fact become almost singular (*i.e.* the determinant is zero) at long periods and thus any inversion for parameters becomes unstable (Groom and Bailey, 1989a). Determination of the long period strike (if there is one) must be done with data from other sites. However, the study indicates that 3D effects are certainly strong at this site and any 2D inversion must be done with care. The major 3D effects are on a quite large scale (*i.e.* the valley, the batholiths).

Conclusions

Although the methodology for determining the dimensionality of the structure governing the MT data works well in synthetic cases, conclusions for field data are sometimes vague. However, synthetic studies have given great insight into the different physical parametrizations and parameter stability. In difficult cases, such as the field example here, multiple sites need to be used to help constrain parameters. These parameters can then be used for modelling and inversion studies.

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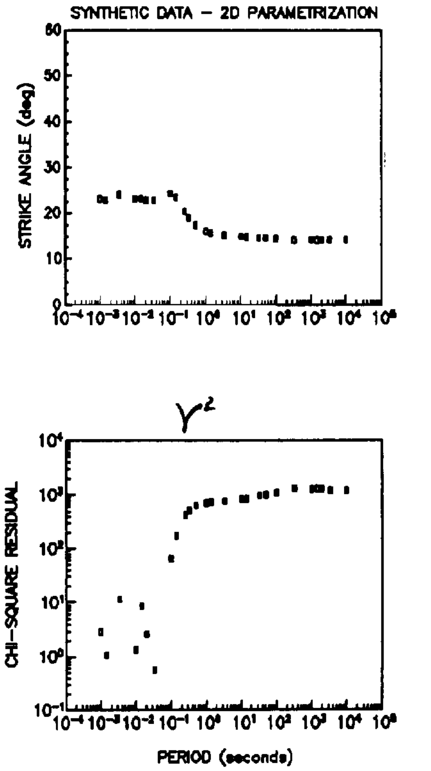

FIG. 1. Conventional 2D strike and the residual error for the conventional two-dimensional parametrization.

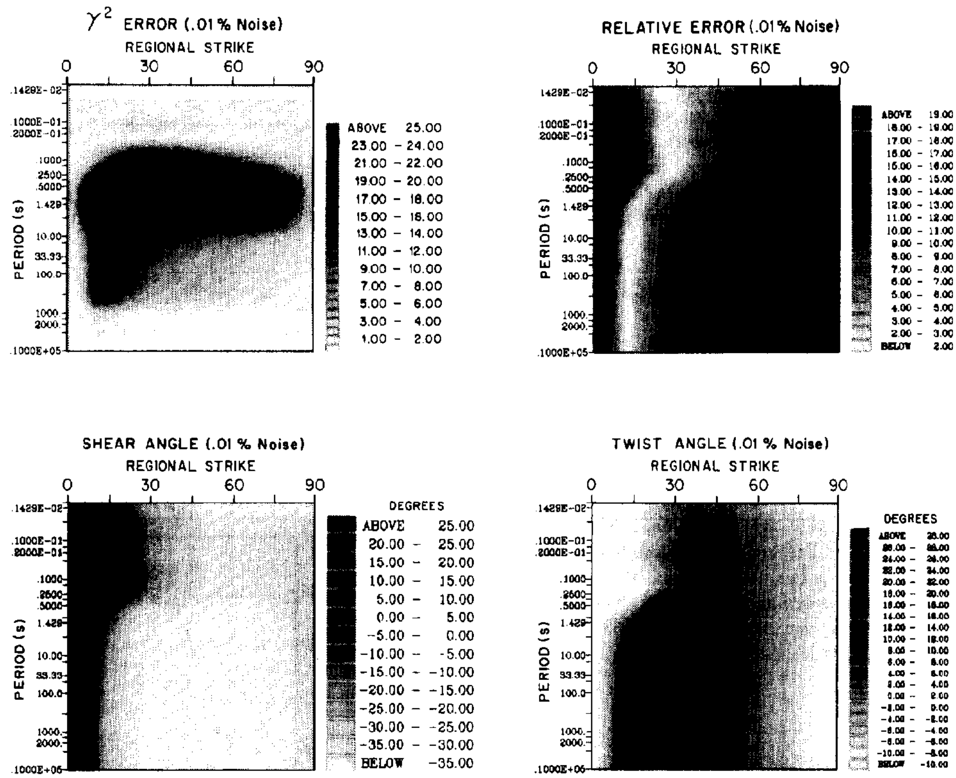

FIG. 2. The residual error, relative error, twist and shear angles for synthetic data with .01% noise.

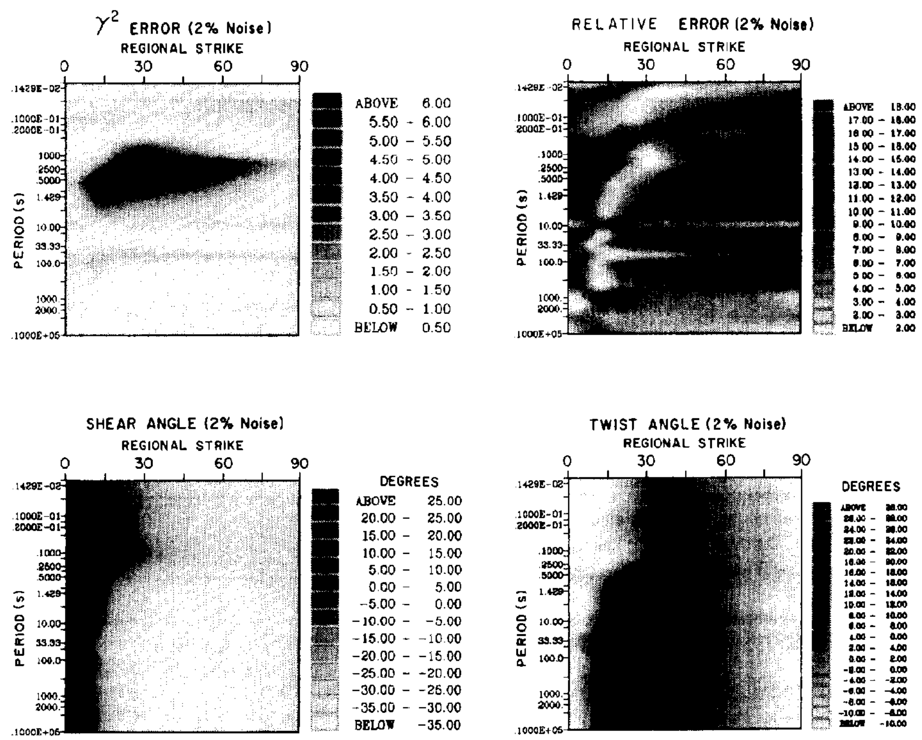

FIG. 3. The residual error, relative error, twist and shear angles for synthetic data with 2% noise.

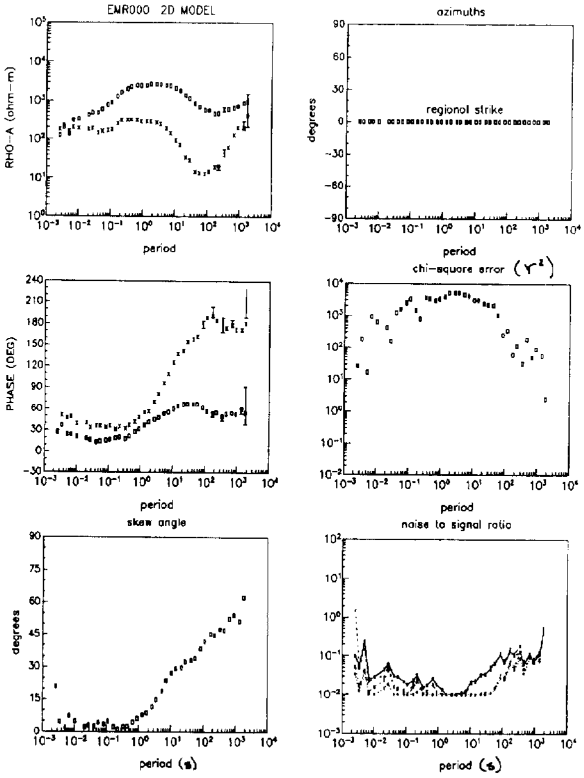

FIG. 4. The 2D model parameters for field data (EMR000) with an assumed N-S strike.

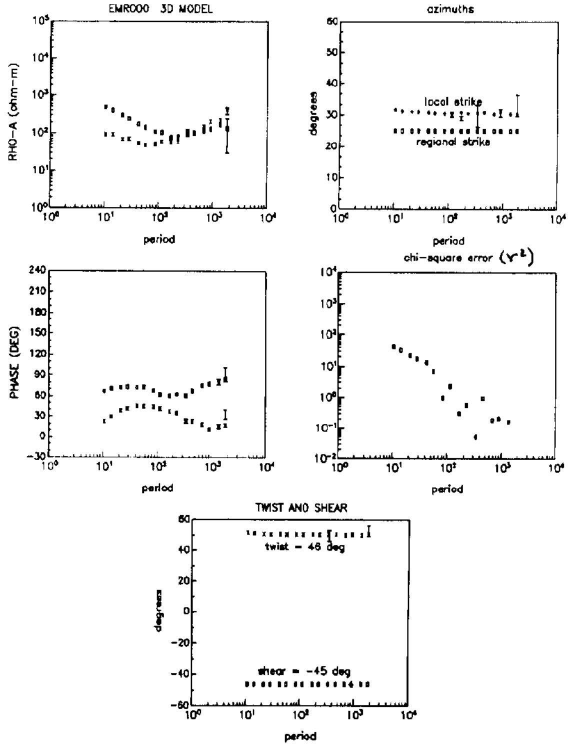

FIG. 6. The 3D model parameters when the regional strike is aligned parallel to the current azimuth.

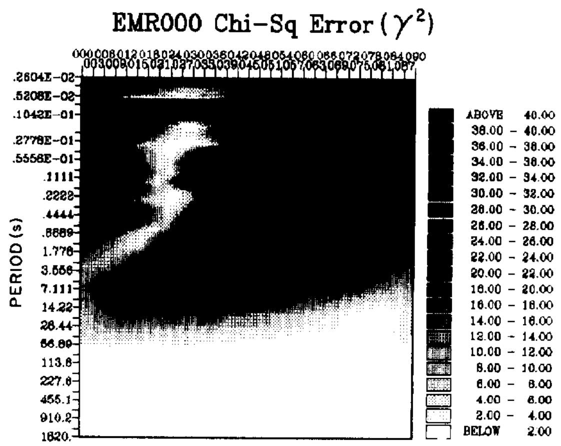

FIG. 5. The chi-square residual for a three-dimensional parametrization of the data from site EMR000 as a function of regional strike and period.